HOFEIM 2011

Workshop on Higher Order Finite Element and Isogeometric Methods

Program and Book of Abstracts

27-29 June 2011

Cracow, Poland

Organized as ECCOMAS Thematic Conference by

Polish Association for Computational Mechanics,
Cracow University of Technology
and Committee on Mechanics of PAS

under auspices of ECCOMAS and IACM

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Local organizing support

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Workshop webpage: `http://hofeim.l5.pk.edu.pl`

Welcome address

In the tradition of the HOFEM workshops organized previously (Bad Honnef, Germany, 1998; St. Louis, USA, 2000; Bad Honnef, Germany, 2003; Herrsching, Germany, 2007), the workshop brings together specialists interested in developing higher-order Finite Element Methods for Partial Differential Equations (PDEs) with applications to Engineering and Science.

This time, the participants of the previous meetings team up with the recently emerged Isogeometric Methods community focusing on extending the use of computational geometry methodologies, such as Non-Uniform Rational B-Splines (NURBS), T-Splines, Subdivision Surfaces, etc., to the solution of PDEs with smooth discretizations. These methods have provided a new direction of research in higher-order finite element methods, and shed light on the accuracy, efficiency and robustness of higher-order finite element methods in general.

Continuing the tradition of the previous meetings, the workshop aims at popularizing the subject of higher-order discretizations to young researchers and graduate students by minimizing the cost of participation. The best contributions will be published in a special issue of Computer Methods in Applied Mechanics and Engineering (CMAME).

The workshop is registered as ECCOMAS Thematic Conference and hosted by Cracow University of Technology in Cracow, Poland. The organization has been supported by the Committee on Mechanics of the Polish Academy of Sciences, Polish Association for Computational Mechanics and two institutes of Cracow University of Technology: Institute for Computational Civil Engineering and Institute of Computer Science.

The workshop is composed of 6 invited review lectures, 24 invited keynotes and 29 poster contributions. The Organizers express their thanks to all Authors of invited lectures and wish all participants a fruitful exchange of scientific ideas as well as a pleasant stay in Cracow.

On behalf of the Organizing Committee
Leszek Demkowicz (Conference Chairman)

Workshop program

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JUNE 26 (SUNDAY)			
6:00 p.m.	Registration and 'get-together'		Canteen, 1^{st} floor, CUT Campus, Warszawska 24
JUNE 27 (MONDAY)			
8:45 - 9:00	Opening		Canteen, room S1, CUT Campus, Warszawska 24
9:00 - 9:50	Plenary talk	**T.J.R. Hughes**	*Isogeometric analysis as a higher-order Finite Element methodology*
9:50 - 10:20	Invited talk	**E. Cohen**	*Representing shapes for both design and isogeometric analysis*
10:20 - 10:50 Coffee break			
10:50 - 11:20	Invited talk	**K.-U. Bletzinger** **J. Kiendl** **R.Wüchner**	*Isogeometric shape optimization of 3D shell structures*
11:20 - 11:50	Invited talk	**L. De Lorenzis** **P. Wriggers**	*Isogeometric analysis of 3D large deformation contact problems*
11:50 - 12:20	Invited talk	**H. Gomez** **Y. Bazilevs** **V. Calo** **T.J.R. Hughes** **X. Nogueira**	*Isogeometric analysis and unconditionally stable time integrators for computational phase-field modeling*
12:20 - 1:30 Lunch			
1:30 - 2:20	Plenary talk	**A. Düester** **E. Rank** **B. Szabó**	*Higher-order Finite Elements for solid mechanics*
2:20 - 2:50	Invited talk	**B. Szabó** **R. Actis**	*FEM in professional practice: the questions of 'what?' and 'how?'*
2:50 - 3:20 Coffee break			
3:20 - 3:50	Invited talk	**Z. Yosibash**	*p-FEMs for biomechanical applications: bones and arteries*
3:50 - 4:20	Invited talk	**E. Rank** **A.Düster** **S. Kollmannsberger** **M. Ruess** **D. Schillinger** **Z. Yang**	*The Finite Cell Method: a higher order fictitious domain approach*
4:20 - 4:50	Invited talk	**I. Páczelt** **Z. Mróz**	*Solution of different wear problems with p-version of Finite Element Method*

5:00 - 6:30 Poster session	
A. Abedian **J. Parvizian** **A. Düester** **E. Rank**	*The Finite Cell Method adaptive integration and application to problems of elastoplasticity*
J. Bramwell **L. Demkowicz** **J. Gopalakrishnan** **W. Qiu**	*A discontinuos Petrov-Galerkin method for linear elasticity*
H. Brandsmeier **K. Schmidt** **Ch. Schwab**	*A multiscale hp-FEM for 2D photonic crystal bands*
M. Bürg	*Towards an hp-adaptive refinement strategy for Maxwell's equations*
N. Collier **V. Calo** **M. Paszyński** **D. Pardo**	*The cost of continuity: a study of the performance of isogeometric Finite Elements using direct solvers*
D. Cho **L. Beirão da Veiga** **L. F. Pavarino** **S. Scacchi**	*Overlapping additive Schwarz methods for isogeometric analysis*
L. Demkowicz **N. Heuer**	*A recipe: how to construct a robust DPG method for the confusion problem (and any linear problem as well)*
W. Dornisch **S. Klinkel**	*Boundary conditions and multi-patch connections in isogeometric analysis*
A. Fröhlcke **E. Gjonaj** **T. Weiland**	*A boundary conformal approach for higher order discontinous Galerkin Finite Element Method*
L. Garcia-Castillo **I. Gomez-Revuelto** **D. Pardo** **J. Kurtz** **M. Salazar-Palma**	*Automatic hp-adaptivity for three dimansional electromagnetic problems. Application to waveguide problems*
I. Jaworska	*On regularization aided HO multipoint solution approach*
L. Kern **A. Schröder** **A. Byfut**	*Constrained approximation in hp-FEM non-matching refinements and multi-level hanging nodes*
L. G. Kocsán **E. Bertóti**	*A two-field dual mixed hp Finite Element Model for cylindrical shells*
R. Kolman **J. Plešek** **M. Okrouhlík** **D. Gabriel** **J. Kopačka**	*B-spline Finite Element response of elastic bar under shock loading*

JUNE 28 (TUESDAY)			
9:00 - 9:50	Plenary talk	**J. Schöberl**	*Hybrid discontinuous Galerkin methods for the Navier-Stokes equations*
9:50 - 10:20	Invited talk	**S. Beuchler** **S. Zaglmayr** **V. Pillwein**	*Sparsity optimization of H(div)- and H(curl)-conforming hp-Finite Elements*
10:20 - 10:50 Coffee break			
10:50 - 11:20	Invited talk	**M. Melenk** **S. Sauter** **M. Löhndorf**	*Wave number-explicit convergence analysis for the Helmholtz equation: hp-FEM and hp-BEM*
11:20 - 11:50	Invited talk	**D. Pardo** **M. Paszyński** **V. Calo** **N. Collier**	*Direct multi-frontal solvers for higher-order Galerkin methods*
11:50 - 12:20	Invited talk	**J. Gopalakrishnan**	*Stable DPG methods with high order approximation spaces*
12:20 - 1:30 Lunch			
1:30 - 2:20	Plenary talk	**A. Buffa**	*Isogeometric vector field approximations: a review*
2:20 - 2:50	Invited talk	**J.A. Evans** **Y. Bazilevs** **I. Babuška** **T.J.R. Hughes**	*On the effectiveness of multi-dimensional and compatible splines in numerical approximation*
2:50 - 3:20 Coffee break			
3:20 - 3:50	Invited talk	**L. Beirão da Veiga** **Y. Bazilevs** **A. Buffa** **A. Cottrell** **D. Cho** **T.J.R. Hughes** **G. Sangalli** **J. Rivas**	*Approximation properties of mapped NURBS spaces for isogeometric analysis*
3:50 - 4:20	Invited talk	**V. Calo** **N. Collier** **L. Dalcin** **M. Knepley** **D. Pardo** **M. Paszyński**	*Application of Isogeometric Finite Elements*
4:20 - 4:50	Invited talk	**A. Reali** **F. Auricchio** **F. Calabrò** **T.J.R. Hughes** **G. Sangalli**	*Quadrature strategies for NURBS-based isogeometric analysis*

5:00 - 6:30 Poster session	
A. Johannessen T. Kvamsdal T. Dokken	*Adaptive refinement in isogeometric analysis using LRB-splines*
S. Kollmannsberger A. Düester E. Rank Ch. Sorger	*To mesh or not to mesh. That is the question*
F. Krużel K. Banaś	*Powerexcell implementation of numerical integration for higher order elements*
A. Niemi N. Collier V. Calo	*DPG method based on the optimal test space norm for steady transport problems*
A. Nowakowski I. Elhadi N. Qin	*Streamline Upwind Petrov Discontinous Galerkin (SUPDG) method for scalar and system conservation laws*
P. Płaszewski P. Macioł K. Banaś	*Numerical integration on GPUs for higher order finite elements*
A. Ratnani E. Sonnendrucker N. Crouseilles	*Isogeometric analysis in plasma physics and electromagnetism*
U. Römer S. Koch T. Weiland	*Shape sensitivity analysis based on isogeometric analysis applied to electromagnetic problems*
G. Sangalli F. Auricchio L. Beirão da Veiga T.J.R. Hughes A.Reali	*Isogeometric collocation techniques for static and dynamic elasticity problems*
C. Scheid S. Lanteri	*Convergence of a discontinous Galerkin scheme for time domain Maxwell's equations in a dispersive media*
S. Shannon Z. Yosibash	*Extracting generalized flux intensity functions along circular singular edges*
I. Soloveichik M. Bercovier	*Additive Schwartz decomposition methods applied to isogeometric analysis*
B. Tóth E. Bertóti	*A three-field dual-mixed hp Finite Element Model for cylindrical shells*
N. Trabelsi Z. Yoshibash	*Reliable patient-specific p-FEM simulation of Femur's mechanical response*
R. Vázquez	*Isogeometric simulation of some real electromagnetic applications*

JUNE 29 (WEDNESDAY)			
9:00 - 9:50	Plenary talk	**T. Kvamsdal** **K.A. Johannessen** **T. Dokken** **K.F. Pettersen**	*Adaptive isogeometric methods*
9:50 - 10:20	Invited talk	**M.A. Scott** **M.J. Borden** **T.J.R. Hughes** **T.W. Sederberg**	*Isogeometric analysis using T-splines*
10:20 - 10:50 Coffee break			
10:50 - 11:20	Invited talk	**J. Zhang** **W. Wang**	*Converting unstructured quadrilateral / hexahedral meshes to T-splines*
11:20 - 11:50	Invited talk	**B. Simeon** **A.-V. Vuong** **C. Gianelli** **B. Jüttler**	*Hierarchical local refinement in isogeometric analysis*
11:50 - 12:20	Invited talk	**Y. Bazilevs**	*Isogeometric analysis of fluid-structure interaction with emphasis on wind turbines*
12:20 - 1:30 Lunch			
1:30 - 2:20	Plenary talk	**P. Gatto** **K. Kim**	*hp-Finite Elements for coupled problems: an overview of our new 3D code*
2:20 - 2:50	Invited talk	**A. Schröder**	*Higher-order Finite Element Methods for contact problems*
2:50 - 3:20 Coffee break			
3:20 - 3:50	Invited talk	**W. Rachowicz** **A. Zdunek** **T. Eriksson**	*Application of hp-adaptive FEM to medical diagnostics*
3:50 - 4:20	Invited talk	**W. Cecot** **M. Serafin**	*Application of adaptive FEM to solution of selected inelastic problems*
4:20 - 4:50	Invited talk	**M. Baitsch**	*Refinement of curvilinear hexahedral meshes for higher order Finite Elements*
7:00 Conference dinner - restaurant ”Wesele”, Main Market Square 10			
JUNE 30 (THURSDAY)			
Social activities (e.g. excursion to Wieliczka salt mine)			

Plenary lectures

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Isogeometric vector field approximations: a review

Annalisa Buffa

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KEYWORDS: isogeometric analysis, exterior calculus

IGA methodologies are designed with the aim of improving the connection between numerical simulation of physical phenomena and the Computer Aided Design systems. This is achieved by using B-Splines or Non Uniform Rational B-Splines (NURBS) for the geometry description as well as for the representation of the unknown fields.

Beside the interoperability of CADs and Analysis, the use of Spline or NURBS functions, together with isoparametric concepts, results in an extremely successfully idea and paves the way to many new numerical schemes for the discretization of PDEs enjoying features that would be extremely hard to achieve within a standard finite element framework.

During this talk, I will mainly review the recent works on the design of Spline spaces which can be used as approximations of differential forms. These spline spaces allow for the construction of discretization schemes which are compatible in the sense that the discretized models embody conservation principles of the underlying physical phenomenon (e.g., charge in electromagnetism, mass and momentum in fluid motion and elasticity).

I will show how these spaces can be used to have isogeometric extensions of “classical” compatible discretizations as Raviart-Thomas elements for Darcy flow, or edge elements for Maxwell equations, but also how regularity of splines can be exploited to extend their applicability to discretize other physical problems such as the Stokes equations for fluids and the Reissner-Mindlin model for plates.

The main references for the results I will be presenting are the following:

REFERENCES

[1] Buffa A., Rivas J., Sangalli G. and Vazquez R.: Isogeometric Discrete Differential Forms in Three Dimensions, *SIAM J. Numer Anal.* , 49, pp 818-844, 2011.

[2] Buffa A., de Falco C. , Sangalli G., Isogeometric Analysis: new stable elements for the Stokes equation *Internat. J. Numer. Methods Fluids*, DOI: 10.1002/fld.2337

[3] Beirao da Veiga L., Buffa A., Lovadina C., Martinelli M., Sangalli G., An isogeometric method for the Reissner-Mindlin plate blending problem, *in preparation.*

[4] J. Evans, Divergence-free B-spline Discretizations for the Stokes and Navier-Stokes Equations, *Ph.D. Thesis*, UT Austin.

HIGH-ORDER FINITE ELEMENTS FOR SOLID MECHANICS

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KEYWORDS: structural/solid mechanics, nonlinear problems, multiscale/multiphysics problems

The presentation will give an overview of the research activities on high-order finite elements which have been conducted during the last couple of years including the current state-of-the-art. The main focus will be on the development and application of high-order finite elements for solid mechanics. Emphasis will be placed on implementational aspects concerning topics like the geometric description of curved elements. Furthermore, the robustness and efficiency of high-order elements will be demonstrated by considering several benchmarks representing applications in structural and mechanical engineering. We will consider problems related to the computation of thin-walled structures, problems of hyperelasticity and elastoplasticity, contact mechanics as well as multiscale and multiphysics problems. Finally we will give an outlook to a new method, combining ideas of high-order finite elements and fictitious domain methods.

REFERENCES

[1] B.A. Szabó, I. Babuška. *Finite element analysis*. John Wiley & Sons, 1991.

[2] A. Düster, H. Bröker, E. Rank. The p-version of the finite element method for three-dimensional curved thin walled structures. *International Journal for Numerical Methods in Engineering*, 52:673–703, 2001.

[3] B.A. Szabó, A. Düster, E. Rank. The p-version of the Finite Element Method. In E. Stein, R. de Borst, T.J.R. Hughes (Eds.): *Encyclopedia of Computational Mechanics*, John Wiley & Sons, Vol. 1, Chap. 5, pp:119–139, 2004.

[4] A. Düster, J. Parvizian, Z. Yang, E. Rank. The finite cell method for three-dimensional problems of solid mechanics. *Computer Methods in Applied Mechanics and Engineering*, 197:3768–3782, 2008.

[5] E. Rank, S. Kollmannsberger, A. Düster. High Order Finite Elements: Principles, Achievements, Open Questions. In B.H.V. Topping et al. (Eds.): Computational Technology Reviews, SAXE-COBURG Publications, 2010.

hp-finite Elements for Couple Problems – an Overview of Our New 3D Code

Paolo Gatto[1], Kyungjoo Kim[2]

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KEYWORDS: hp-finite elements, shape functions, deadlock, parallel solver.

In this talk we review the fundamental steps needed to build an hp-code (choice of element shapes, construction of shape functions, p-enrichments, h-refinements, solver) and focus on the implementation that we have developed in our research group led by Professor Leszek F. Demkowicz at The University of Texas at Austin.

Our latest in-house code, *hp3d*, was built to simulate coupled multi-physics problems, i.e., the variables of interest may belong to different energy spaces, through variable-order, exact-sequence finite elements of all shapes—tetrahedra, hexahedra, prisms, and pyramids as well—. Allowing for elements of all shapes is crucial for successfully modeling thin-walled structures. The physical variables are related through weak couplings. On the theoretical side, this guarantees existence of the solution for the continuous problem, while on the implementation side it allows for a rescaling of the physical quantities across the interfaces.

Variable order elements call for a construction of hierarchical H^1, $H(\mathrm{curl})$, $H(\mathrm{div})$-conforming shape functions for 1D (edge), 2D (triangle, quad) and 3D elements. The logic of implementation is based on identifying a set of core (kernel, bubble) 1D and 2D functions, and the use of specific edge-to-element and face-to-element extensions. The dependence of 3D shape functions on different edge and face orientations is taken into account at the level of element shape functions routine. This simplifies dramatically the assembly procedure and the implementation of constrained approximation allowing for the presence of hanging nodes. The construction of H^1 shape functions is directly related to transfinite interpolation techniques used in *Mesh Based Geometry* (MBG) descriptions that constitutes the foundations of our Geometry Modeling Package.

A consistent departure from previous implementations is in the data structure. Both unconstrained and constrained nodes are explicitly stored in the data structure and their status is recorded through a flag. This choice is motivated by the refinement algorithm. In simple terms, first we break an element of choice, then we perform additional refinements to recover a 1-irregular mesh (only 1 level of hanging nodes is allowed) suitable for computations. As a consequence of this approach, the data structure needs to support meshes with lower regularity then 1-irregular. The refinement algorithm has been designed in order to avoid *deadlocks*; we have experimental results that this is indeed the case.

We also present a highly scalable parallel sparse direct solver on the multi-core architectures using Un-assembled Hyper Matrices(UHM) for the problems arising from $hp-$adaptive Finite Element Methods. Our scheme consists of storing the matrix as unassembled element matrices, hierarchically ordered by mirroring the refinement history of the domain or recursive Domain Decomposition. The hierarchical structure of unassembled matrices and independent local data storage naturally leads to the task parallelism via the Divide and Conquer algorithm. As the second level of task parallelism, *algorithms-by-blocks* is exploited to achieve higher scalability. We compare the performance against other sparse direct solver packages on the 24 core machine.

ISOGEOMETRIC ANALYSIS AS A HIGHER-ORDER FINITE ELEMENT METHODOLOGY

Thomas J.R. Hughes[1]

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I will describe how Isogeometric Analysis may be viewed as a higher-order finite element methodology and how it may be easily implemented in existing finite element computer programs through a shape function subroutine. I will review current developments in the Isogeometric Analysis [1,2] approach to problems of computational mechanics and present recent progress toward developing integrated CAD/FEA procedures that do not involve traditional mesh generation and geometry clean-up steps, that is, the CAD file is directly utilized as the analysis input file. I will also summarize some of the mathematical developments within Isogeometric Analysis that confirm the superior accuracy and robustness of spline-based approximations compared with traditional FEA and expand the scope of applications to new areas. Sample applications will be selected from problems of linear and nonlinear solids and structures, and fluids and fluid-structure interaction.

REFERENCES

[1] T.J.R. Hughes, J.A. Cottrell and Y. Bazilevs. Isogeometric Analysis: CAD, Finite Elements, NURBS, Exact Geometry and Mesh Refinement. *Computer Methods in Applied Mechanics and Engineering. Vol. 194, Nos. 39-41, pp. 4135-4195*, 2005.

[2] T.J.R. Hughes, J.A. Cottrell and Y. Bazilevs. Isogeometric Analysis: Toward Integration of CAD and FEA. Wiley, Chichester, U.K., 2009.

ADAPTIVE ISOGEOMETRIC METHODS

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KEYWORDS: Isogeometric analysis, Adaptive methods, LR B-splines

We will start out by presenting an overview of the current research activities related to error estimation and adaptivity for isogeometric finite element methods.

The focus will then be on adaptive refinement using LR B-splines pioneered by Tor Dokken. LR B-splines allow for local refinement as T-joints and contains refining algorithms which exactly preserve the geometry [1].

The adaptive procedures will be tested on benchmark problems with known analytical solution.

REFERENCES

[1] T. Kvamsdal, K. A. Johannessen and T. Dokken. Adaptive Isogeometric methods using LR B-splines. In *MekIT'11 Sixth National Conference on Computational Mechanics*, Editors B. Skallerud and H. I. Andersson, Page 157–170, 2011, Tapir Academic Press, Trondheim, ISBN:978-82-519-2798-7.

Hybrid discontinuous Galerkin methods for the Navier-Stokes equations

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Discontinuous Galerkin finite element methods provide a lot of freedom to obtain desired stability properties of numerical schemes. In particular, the upwind choice of numerical fluxes allow large convective terms, e.g. large Reynolds numbers. Furthermore, by relaxing the continuity constraints of the finite element basis functions it becomes simple to construct exactly divergence free discrete approximations leading to stability in kinetic energy.

Discontinuous Galerkin methods lead to an increased number of unknowns, and even worse, to a much stronger coupling in the stiffness matrix. Here, recent hybridization techniques come into the game. One introduces even more unknowns on element interfaces. But now, the coupling between elements is reduced to the interface variables, and the element unknowns can be eliminated by static condensation.

In our talk we discuss the construction of such hybrid DG methods for the incompressible Navier-Stokes equations. We discuss the connection to time integration (in particular splitting methods), and iterative solvers (in particular domain decomposition methods). Numerical results for benchmark problems are presented.

Keynote lectures

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REFINEMENT OF CURVILINEAR HEXAHEDRAL MESHES FOR HIGH ORDER FINITE ELEMENTS

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KEYWORDS: template-based mesh refinement, composition of geometry mapping, shape optimization

In this paper, a method for the conforming refinement of high-order hexahedral meshes on curvilinear domains is presented. Two basic problems are addressed: How to establish the geometrical mapping of the refined elements and how to determine a combination of refinement templates such that the existing elements can be split without generating hanging nodes.

The presented approach uses a catalogue of refinement templates (see e.g. [1]) for faces and elements. Individual patterns are identified by the number of nodes inserted on edges, whereas currently, a maximum number of two nodes per edge is allowed. The refinement patterns, which are defined in a reference orientation, can be applied to the actual situation by transforming the geometry and the topological relations.

During the refinement of a single element according to a certain refinement template, the geometry mappings of the new elements are established by reusing the original functions: The template along with the node positions on the edges define a mesh of elements $\tilde{K}_i$ on the reference domain $\hat{K}$. For each element $\tilde{K}_i$ on the reference domain, the geometry map $\tilde{\mathbf{Q}}_i : \hat{K} \rightarrow \tilde{K}_i$ is used to construct the geometry functions $\mathbf{Q}_i$ of the new elements as composition $\mathbf{Q}_i = \mathbf{Q}_o \circ \tilde{\mathbf{Q}}_i$ where $\mathbf{Q}_o$ is the geometry of the original element, see Figure 1. A similar procedure is applied in order to refine edges and faces.

The actual refinement of a set of elements is carried out by an algorithm which takes a collection of new nodes placed on the edges to be refined as input data. Based upon this input, an unrefined element, for which a unique choice of a refinement template exists, is selected. According to the refinement template, new nodes are inserted on the affected edges (if required) and the new elements are created. These steps are repeated until all elements in the input set are refined. In ambiguous cases, the algorithm can generate all possible solutions, however, since this is a combinatorial problem, it is most often advisable to specify additional edge refinements.

The template-based algorithm considerably simplifies mesh refinement on curvilinear domains. Since only a small amount of input data is required, this approach is well suited for interactive applications where the mesh is refined a priory along geometrical features such as inclined edges. The repeated application of the procedure allows it to generate multiple refinement layers easily. Another application area is shape optimization, where the geometry of refined elements is automatically updated upon changes in the design.

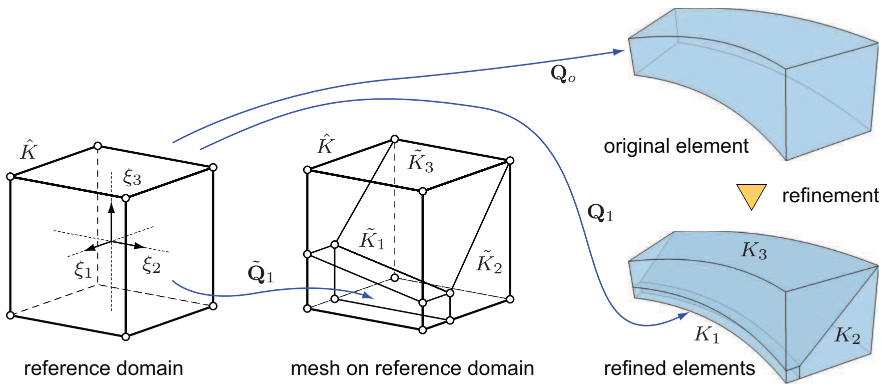

Figure 1: Refinement of a curvilinear element

References

[1] Dae-Young Kwak and Yong-Taek Im. Remeshing for metal forming simulations—Part II: Three-dimensional hexahedral mesh generation. *International Journal for Numerical Methods in Engineering*, 53(11):2501–2528, 2002.

ISOGEOMETRIC ANALYSIS OF FLUID-STRUCTURE INTERACTION WITH EMPHASIS ON WIND TURBINES

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KEYWORDS: Isogeometric Analysis, NURBS, T-Splines, Wind Turbine Blades

Isogeometric Analysis (IGA) [1], despite its young age, has significantly matured as a technology for geometry representation and computational analysis. Although NURBS remain the most popular means of geometry modeling for IGA, advances in T-Spline and Subdivison surface representations enabled the solution of computational problems requiring local mesh refinement, which is not easily accomplished with NURBS. Advances in model quality definition and improvement enabled the generation of better- parameterized shapes for IGA, thus improving the quality of the computational solution. Recent efforts to define standardized file formats for data exchange between the geometry modeling and computational analysis software enabled straightforward solution of complicated structural problems that involve large deformation, plasticity and contact, using well-validated commercial FEM software. Furthermore, IGA is able to superbly handle many applications that otherwise create significant challenges to standard finite element technology.

However, many challenges remain for IGA to be fully accepted as an industrial-grade analysis technology. The ability to create 3D volumetric complex geometry models in an automated manner is one such challenge. Another challenge is to prove that IGA is capable of producing accurate and robust results for complex-geometry multi-physics problems (e.g., fluid-structure interaction), which is one of the major demands of modern computational analysis.

This presentation will focus on the coupling strategies, specific to IGA, for multi-physics applications that make use of non-matching descriptions of geometry at the interface between different physical subsystems. These coupling procedures allow greater flexibility in the computational analysis, and, simultaneously, alleviate the difficulties of geometry modeling and construction of interfaces that match parametrically. Applications to problems of engineering interest will be shown. In particular, applications to wind turbine simulation will be presented in great detail.

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APPROXIMATION PROPERTIES OF MAPPED NURBS SPACES FOR ISOGEOMETRIC ANALYSIS

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KEYWORDS: Isogeometric Analysis, approximation estimates

In this contribution we review the approximation properties of the discrete spaces uses in Isogeometric Analysis, introduced by T.J.R. Hughes and co-workers in 2005.

Isogeometric Analysis (in short IGA) follows the isoparametric paradigm in the geometric framework described by CAD. In other words, in order to (1) be able to exactly describe CAD type geometries and (2) follow an isoparametric approach, IGA adopts a discrete space which is generated by mapped NURBS basis functions. Such functions are piecewise rational polynomials, mapped from a standard parametric space into the physical geometry.

Independently of the particular IGA numerical scheme that is used, be it for instance a Galerkin or a collocation discretization, a fundamental condition for the convergence of the method is the approximation properties of the discrete space. In this contribution we investigate the recent IGA literature on the approximation properties of mapped NURBS. Such literature, which takes the steps from the approximation properties of splines (see for instance the Shumaker and De Boor books), starts in 2006 with contribution [1].

In paper [1], among other results, the authors show an h-approximation analysis, which makes use of a generalized Bramble-Hilbert lemma in bent Sobolev spaces in order to overcome difficulties which are peculiar to the NURBS framework. More recently, in [2], a different approach more related to *hp* type analysis is instead followed, with the scope of studying the behavior of the NURBS approximation also with respect to k and p. The authors are able to give precise results, but only in the case $k \lesssim p/2$, which allows for certain fundamental simplifications. Finally, in [3] the authors concentrate again on the h-analysis, in this case in order to obtain results which apply also to anisotropic estimates. The adopted arguments are again different, and are essentially based on a one dimensional estimate.

During the talk, all the shown theoretical results will be supported by a range of numerical tests.

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Isogeometric shape optimization of 3D shell structures

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Keywords: isogeometric, shape optimization, shell

Shape optimization aims at finding the optimal shape of a structure with respect to a specific objective. Typical objectives are structural properties like maximum stiffness, minimum weight, etc. For shells, and especially for thin shells, the overall structural behaviour is crucially determined by their shape. An optimal shape for minimizing the weight, for example, carries all loads by membrane forces and no bending moments appear, which guarantees an efficient use of the material.
Applying the isogeometric concept to shape optimization, the distinction between CAD-based and FE-based optimization is redundant since both models rely on the same geometric basis. Therefore, the advantages of both approaches can be combined, and furthermore, the whole process of design, analysis and shape optimization can be integrated into one geometric model. Nevertheless, it is important to carefully distinguish between analysis and optimization model and the various levels of intermediate refinements of the latter. This approach is presented for shape optimization of thin shells, based on isogeometric shell analysis as presented in [1,2].
Besides the promising methodological aspects of IGA based shape optimization the paper will discuss several principal points of shape optimal design of shell structures, as there are: the properties of large, non-convex design spaces, the non-uniqueness of shape parameterization, and how to treat them, as well as the utility and futility of optimal solutions which IGA based optimization shares with other techniques.

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Applications of Isogeometric Finite Elements

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Since November we have been working on fast implementations of B-spline/NURBS based finite element solvers, written using PETSc. PETSc is frequently used in software packages to leverage its optimized and parallel implementation of solvers, however we also are using PETSc data structures to assemble the linear systems. These structures in PETSC (called DAs) were originally intended for the parallel assembly of linear systems resulting from finite differences. We have reworked this structure for linear systems resulting from isogeometric analysis based on tensor product spline spaces. The result of which is a framework for solving problems using isogeometric analysis which is scalable and greatly simplified over previous solvers. Several applications of this framework to linear and nonlinear model problems will be presented and simulation results described.

APPLICATION OF ADAPTIVE FEM TO SOLUTION OF SELECTED INELASTIC PROBLEMS

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KEYWORDS: hp-adaptive FEM, shakedown, elastic-visco-plasticity, homogenization

The adaptive finite element method has been widely used in numerical analysis of solid mechanics problems in elastic range. However, also inelastic analysis, mainly in the framework of deformation theory of plasticity, was performed by either h or $p-$adaptive versions of FEM [4]. Some of the authors report superior performance of the $p-$adaptive FEM in comparison with the h one (e.q. [3]) in such cases. We have applied the h and $hp-$adaptive FEM [2] to shakedown problems, elastic-visco-plastic analysis [1] as well as modeling of heterogeneous materials by computational homogenization [5], where the $hp-$adaptive FEM is used at two scales with flow plasticity theory as the constitutive model at the microlevel.

Application of the adaptive mesh refinement required studying such numerical issues as: appropriate a'posteriori error estimation both in space and time, transfer of the solution from an old mesh to a new one or strategy of mesh adaptation. The multiscale analysis includes also assessment of the modeling error that results from the computational homogenization.

Our numerical results of residual stress computation by both direct shakedown approach and more precise elastic-visco-plastic analysis, reconstruction of residual stresses in a plate on the basis of experimental measurements by inverse problem solution, as well as modeling of elastic-plastic deformations of metal matrix composites using the computational homogenization confirm good performance of the adaptive, in particular $hp-$adaptive FEM, for inelastic problems.

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Representing Shapes for both Design and Isogeometric Analysis

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Isogeometric Analysis (IA) has been proposed as a methodology for bridging the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA). While CAD typically focuses on a boundary representation, FEA has focused on volumetric representations. Creating high quality representations for just one of these goals can be challenging. However, proposed representations for IA must create parameterizations and elements suitable for supporting both good geometric computations and have good qualities for this new mode of analysis. Further different analysis methods have wanted different properties in their meshes. This presentations discusses some of the challenges in moving from current representational and modeling methodologies, and some initial parameterization, modeling/reconstruction methodologies towards creating models that satisfy both domains.

ON THE EFFECTIVENESS OF MULTI-DIMENSIONAL AND COMPATIBLE SPLINES IN NUMERICAL APPROXIMATION

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KEYWORDS: splines, approximation, isogeometric analysis, n-width, sup-inf, variational eigenproblem, Lanczos iteration

Since its introduction in 2005 by Hughes, Cottrell, and Bazilevs [1], isogeometric analysis has emerged as a popular design-through-analysis technology. In isogeometric analysis, the basis which is used to describe computer aided geometry is also utilized for finite element analysis. Typically, such a basis composes of polynomial or rational splines. Hence, a natural question arises as to the effectiveness of splines in numerical approximation and, ultimately, finite element analysis. In this talk, we assess the effectiveness of multidimensional splines as approximating functions utilizing the theory of Kolmogorov n-widths. Numerical algorithms for computing n-widths and sup-infs in a Sobolev space setting are presented based on the solution of two variational eigenproblems. These algorithms result from the application of Galerkin's method and Lanczos iteration. A numerical study is conducted in which we compute the n-width and sup-inf for a large class of smooth multi-dimensional splines. This study reveals the near-optimal approximation properties of smooth multi-dimensional spline functions as well as smooth compatible spline functions satisfying a discrete de Rham diagram. We finish this talk by comparing the approximability of multidimensional splines exhibiting maximal continuity with classical nodal and edge finite element basis functions.

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ISOGEOMETRIC ANALYSIS AND UNCONDITIONALLY STABLE TIME INTEGRATORS FOR COMPUTATIONAL PHASE-FIELD MODELING

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KEYWORDS: Isogeometric analysis, phase field, unconditionally stable

Phase-field models typically involve higher-order partial-differential operators which are difficult to deal with by standard finite element approaches that utilize $\mathcal{C}^0$ trial and weighting functions. Our approach is based on Isogeometric Analysis, permitting simple and efficient discretizations through the use of continuously differentiable splines.

We present results for the Cahn-Hilliard equation, a two-phase model applicable to the segregation of phases in binary alloys. We are able to compute mesh independent, equilibrium solutions in two and three dimensions through the use of an adaptive time-stepping strategy and a local renormalization of the Cahn-Hilliard parameter that governs the thickness of diffuse interface layers. We have also applied our methodology to the Navier-Stokes-Korteweg equations, which describe water/water-vapor two-phase flow. We present solutions involving condensing vapor bubbles in two and three dimensions

Time permitting, we will present our new unconditionally stable mixed variational method for the phase-field crystal equation, a phase-field model that describes the microstructure of two-phase liquid-solid systems. Our algorithm requires the use of $\mathcal{C}^1$ basis functions, and we employ again discretizations based on continuously differentiable splines. To the best of our knowledge, this is the first second-order time accurate unconditionally stable method for the phase-field crystal equation.

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STABLE DPG METHODS WITH HIGH ORDER APPROXIMATION SPACES

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There are compelling reasons to approximate solutions of partial differential equations using high order polynomials, NURBS, plane-waves etc. However, once we choose such a subspace of approximating functions (the trial space), it is often not easy to establish stability within a ”mixed” framework. In a mixed framework, one tries to approximate the solution as well as its fluxes simultaneously. Traditionally, stability is achieved by carefully balancing the approximation spaces for the solution and the fluxes, but this is a difficult task even for standard high order polynomials spaces, not to mention the many other novel trial spaces that researchers are interested in these days.

In this talk, we will review a class of new discontinuous Petrov-Galerkin (DPG) methods which obtain stability via a different approach. Given any (standard or non-standard) trial space, we will show how one can locally and automatically construct a test space that guarantees stability.

Numerical examples illustrating the extraordinary stability of these new methods will be shown. Stability with respect to variations in h (mesh size) and p (polynomial degree) can be theoretically established for several multidimensional boundary value problems. The robustness of the method when applied to elasticity is evident numerically and is provable using a few new theoretical techniques. In wave propagation problems, we numerically observe robustness with respect to wavenumber (resulting in solutions with hardly observable phase errors).

The talk aims to convey the potential of these methods through a few chosen numerical examples and simple but far-reaching theoretical justifications.

ISOGEOMETRIC ANALYSIS OF 3D LARGE DEFORMATION CONTACT PROBLEMS

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KEYWORDS: contact, isogeometric analysis, large deformation

Isogeometric analysis of contact problems has a good potential to yield significant advantages with respect to the use of standard Lagrange discretizations. As NURBS geometries can attain the desired degree of continuity at the element boundaries, they possess the premises to alleviate all the problems arising particularly in sliding contact when using conventional Lagrange polynomial elements, which are only C0-continuous at the interelement nodes. Such problems have often been faced by introducing smoothing techniques, some of which involving NURBS interpolation. These procedures generally improve the performance of the contact algorithms by enhancing the continuity of the contact surfaces, however they do not increase the order of convergence as the higher-order approximation does not involve the bulk behavior of the solids.

In this work, NURBS-based isogeometric analysis is adopted to model large deformation 2D frictional and 3D frictionless contact problems. The proposed contact formulation is based on a mortar approach, extended to NURBS-based interpolations, and combined with a simple integration procedure which does not involve segmentation of the contact surfaces. Both the penalty and the augmented Lagrangian methods are formulated and implemented.

The presented examples deal with both small- and large-deformation cases. The quality of the solution is examined in terms of contact stress distributions in the small-deformation examples, and in terms of global load vs. displacement behavior for the large-deformation, large-sliding examples. In both cases, the results obtained with the isogeometric analysis and with the traditional Lagrange discretizations are compared for varying resolution and order of the contact surfaces.

Based on results obtained in this investigation, it can be concluded that the proposed contact mortar formulation using NURBS-based isogeometric analysis displays a significantly superior performance with respect to the same formulation using standard Lagrange polynomials. This superiority is a combined effect of the higher continuity achieved at the inter-element boundaries and of the inherent non-negativeness of the NURBS interpolation functions. While these two favorable features may also be individually obtained in different ways (e.g., higher geometric continuity can be pursued by means of smoothing techniques and inherent non-negativeness is possessed by other categories of shape functions), NURBS-based isogeometric analysis provides a very simple framework in which both are simultaneously and naturally achieved.

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Wavenumber-explicit convergence analysis for the Helmholtz equation: hp-FEM and hp-BEM

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KEYWORDS: Helmholtz equation, stability of discretizations

We consider boundary value problems for the Helmholtz equation at large wave numbers k. In order to understand how the wave number k affects the convergence properties of discretizations of such problems, we develop a regularity theory for the Helmholtz equation that is explicit in k. At the heart of our analysis is the decomposition of solutions into two components: the first component is an analytic, but highly oscillatory function and the second one has finite regularity but features wavenumber-independent bounds.

This understanding of the solution structure opens the door to the analysis of discretizations of the Helmholtz equation that are explicit in their dependence on the wavenumber k. As a first example, we show for a conforming high order finite element method that quasi-optimality is guaranteed if (a) the approximation order p is selected as p = O(log k) and (b) the mesh size h is such that kh/p is small. As a second example, we consider combined field boundary integral equation arising in acoustic scattering. Also for this example, the same scale resolution conditions as in the high order finite element case suffice to ensure quasi-optimality of the Galekrin discretization.

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SOLUTION OF DIFFERENT WEAR PROBLEMS WITH *P*-VERSION OF FINITE ELEMENT METHOD

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KEYWORDS: Sliding wear, steady wear state, variational principle, *p*-version of FEM, heat generation

The wear process on the frictional interface of two bodies in a relative sliding motion induces shape evolution. Usually the simulation of the contact shape evolution is provided by numerically integrating the modified Archard wear rule expressed in terms of relative slip velocity and contact pressure. (The modified Archard wear rule specifies the wear rate in normal direction to the contact surface). However, much more effective procedure was shown in paper [1] that the minimization of the total wear dissipation power at the contact interface specifies the steady wear regime. The optimality conditions of the functional provide the contact stress distribution and the wear rate of the rigid body motion. It is important to note that in general contact conditions the vectors of wear rate is not normal to the contact surface and has tangential component. A fundamental assumption is now introduced, namely, *in the steady state the wear rate vectors of bodies are collinear with the rigid body wear velocity allowed by the boundary conditions.*

The Coulomb dry friction conditions and the tangential slip rule are assumed. It is assumed that the displacements and deformations are small, the materials of the contacting bodies are linearly elastic. The discretization of the contacting bodies was performed by the displacement based on *p*-version of finite elements [2] assuring fast convergence of the numerical process and accurate specification of geometry for shape optimization.

It is proved that the derived formulae for contact pressure distribution are independent of the relative sliding velocity and some wear parameters. The formulae for contact pressure distribution are also derived when the wear parameters depend on temperature. Minimization of the wear functional with equilibrium constraints for body B_1 gives results for steady state wear process of arbitrary shape of contact surface. The non-linear equations can be solved by applying the Newton-Raphson technique.

In the thermo-elastic problem for fixed contact zone a steady wear state is reached for which the contact stress distribution in a moving body B_1 (punch) is fixed and moves with the contact zone translating along the body B_2. The first specific case is related to wear analysis induced by a punch translating on an elastic strip. The second example is related to a drum braking system (for different support conditions) for which the stationary pressure distribution is not constant but corresponds to steady wear state. To generate non-oscillating solutions of temperature fields the streamline upwinding (Petrov-Galerkin) formulation can be applied in the presented examples. It is shown that the thermal distortion affects essentially the optimal contact shape.

Some examples demonstrate that the wear process proceeds in a steady state when the numerical process starts at shapes corresponding to the optimal solution. This procedure gives a good possibility to control the error of finite element solution and convergence.

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DIRECT MULTI-FRONTAL SOLVERS FOR HIGHER-ORDER GALERKIN METHODS

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KEYWORDS: *hp* finite element method, multi-frontal direct solver, parallel computations, out-of-core

Solving a system of linear equations is typically the dominating cost of most high order finite elements and isogeometric methods. In here, we focus on the use of direct solvers since they are are essential for a large number of applications and, in addition, they frequently constitute the main building block of iterative solvers such as multigrid. Specifically, we consider multifrontal direct solvers, which are ”smart” implementations of the well-known LU factorization algorithm for sparse matrices. The presentation is divided into three parts:

1. First, we estimate theoretically the CPU time cost and memory usage of a multifrontal solver in terms of the problem size, polynomial order of approximation, and regularity of the approximated solutions. From these estimates, we conclude that additional regularity in the approximated solution increases dramatically the cost of the solver.

2. Second, we employ a well-known solver (MUMPS) to compare the theoretical estimates versus numerical results both for *hp*-FEM as well as for IGA. Numerical results agree with the theoretical ones.

3. Finally, based on the above estimates and the numerical results, we describe a new parallel in-core and out-of-core multifrontal solver that aims to provide superior performance. This solver is designed to deal efficiently with different types of high-order Galerkin methods, including *hp*-FEM, IGA, Fourier-Finite-Element methods, and multi-physics applications.

Exemplary applications are multi-physics computations of the acoustic of the human head [1] and borehole resistivity measurements simulations in deviated wells [2]. The first problem implies the necessity of utilizing different number of degrees of freedom per node. Moreover, it is necessary to use global p adaptive scheme resulting in higher order polynomials over the *hp* finite elements. The second problem results from application of the Fourier series expansion for the solution in the azimuthal direction over the two dimensional mesh. It implies the number of equations equal to the number of Fourier modes utilized. The *hp* adaptive algorithm generates the two dimensional mesh with highly non-regular distribution of degrees of freedom. Both these problems are solved by using parallel multi-frontal direct solver. The first problem is a fully three dimensional problem and it requires an out-of-core feature to be solved [3], while the second problem can be solved in core [4].

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APPLICATION OF *hp*-ADAPTIVE FEM TO MEDICAL DIAGNOSTICS

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KEYWORDS: soft tissue, bioelectromagnetics, biomechanics, HO-FEM, *hp*-adaptivity

The presentation consists of two parts. The first part concerns the use of the *hp*-version of the Finite Element Method (FEM) with adaptivity for medical diagnostics, for example for tumour detection, by solving electromagnetic inverse scattering problems, while the other part is devoted to the accurate and reliable prediction of the mechanical response of soft tissue, especially arteries with aneurysms, or with atherosclerostic changes, that lead to a high risk for rupture when pressurized.

In the first part we present a technique to solve inverse medium scattering problems in electromagnetics. Our motivation is to develop a method of microwave tomography capable to create three-dimensional images of biological objects based on reconstruction of distribution of complex electric permittivity. The method might find applications in medicine and other areas. As advantages of the approach one might consider its non-invasive character (no ionisation) and high contrast of tissues affected by a possible medical condition. The reconstruction of distribution of electric permittivity is obtained by solution of the inverse medium scattering problem. We try to minimize the misfit between the measured waves scattered by the object due to its illumination by incident waves, and the simulated scattered waves corresponding to trial distributions of electric permittivity. The simulations are performed with FEM, and their accuracy must be sufficient at least to match the accuracy of the measurements. This is done by applying *p*-adaptivity of the FE-mesh, i.e. enriching order of elements in the area of large errors. They show up especially in regions with rapid changes of material parameters and in the vicinity of transmitters radiating the illuminating waves. The method based on solving the inverse scattering problem is able to reconstruct the distributions of electric permittivity with perturbations localized to small areas which raises hopes for its possible practical applications.

In the second part arteries are considered as tube-like structures built-up of layers consisting of nearly incompressible rubberlike matrix materials reinforced by collagen fibers [1]. Collagen fibers are known to become nearly inextensible at large strains. We present results from an on-going investigation into the efficiency and robustness of the *hp*-version FEM with *p*-adaptivity in domains with smooth nearly-isochoric deformations and with *h*-adaptivity, i.e. by subdividing elements, in cases involving strong stress gradients due to reinforcement by strongly anisotropic nearly inextensible fibres. The ability of the higher-order FEM to avoid volumetric locking and locking due to strong anisotropy due to the presence of fibers with a limited extensibility is investigated and presented. Volumetric locking is avoided using a generalisation of the two-field formulation of displacement-pressure type [2] with no interelement continuity enforced for the pressure variable.

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THE FINITE CELL METHOD: A HIGH ORDER FICTITIOUS DOMAIN APPROACH

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KEYWORDS: Finite Cell Method, fictitious domain, high order finite elements

The Finite Cell Method (FCM) combines high order Ansatz spaces with a fictitious domain approach. The arbitrarily shaped domain of computation is embedded in a larger, simply shaped region which can easily be meshed in, e.g., a grid of rectangular cells. The method was first investigated for 2D- and 3D-problems in linear elasticity, where it was shown that excellent accuracy and even an exponential p-convergence can be obtained for smooth problems. This presentation will give an overview on extensions of the FCM to geometrically and physically non-linear problems as well as on adaptive refinement techniques. It will discuss issues of an efficient implementation and show, how FCM can be embedded in a *computational steering* framework for an interactive simulation of a total hip replacement. Finally, the close connection to isogeometric analysis will be pointed out. It will be demonstrated how the combination of FCM and IGA offers an easy and efficient possibility for a structural simulation of trimmed surfaces.

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QUADRATURE STRATEGIES FOR NURBS-BASED ISOGEOMETRIC ANALYSIS

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KEYWORDS: Isogeometric Analysis, NURBS, B-Splines, Quadrature

In the framework of NURBS-based isogeometric analysis (see, e.g., [1-2]), an issue to be definitely considered is the study of efficient quadrature techniques, to be used instead of classical Gauss rules (which are far from being optimal in the case of high inter-element smoothness).

In this context, a rule of thumb emerges, i.e., the half-point rule, indicating that optimal rules involve a number of points roughly equal to half the number of degrees-of-freedom, or, equivalently, half the number of basis functions of the space under consideration (see [3]). The half-point rule is independent of the polynomial order of the basis.

In general, efficient rules require taking into account the precise smoothness of basis functions across element boundaries. Following this idea, new rules of practical interest are obtained and shown in this work. Numerical tests are presented in order to prove the efficiency of the new rules as compared with standard Gauss quadrature.

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HIGHER-ORDER FINITE ELEMENT METHODS FOR CONTACT PROBLEMS

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KEYWORDS: contact problems, error estimation, higher-order fem

In this talk, finite element methods of higher-order are presented for contact problems in elasticity. The discretization approach relies on a saddle point formulation where the introduced Lagrange multiplier is defined on the surface of one of the bodies in contact. This approach was originally proposed by Hlávaček et al. [1] for low-order finite elements and is extended to higher-order discretizations, cf. [2,3].

To guarantee the stability of the mixed scheme, a uniform discrete inf-sup condition for the higher-order approach is verified. It is shown that the discrete inf-sup condition is fulfilled if the quotients of the mesh sizes and the polynomials degrees are suitably small. For low-order finite elements of the proposed type, the discretization of the Lagrange multipliers necessitates boundary meshes with a different mesh size than that of the primal variable. In the higher-order approach, this assumption can, in principle, be avoided using different polynomial degrees.

Assuming the discrete inf-sup condition, the convergence of the scheme and some a priori estimates are proven where the definition of the Lagrange mulitpliers in Gauss quadrature points is exploited. Additionally, a posteriori estimates are presented which include the discretization error of an auxiliary problem and some further terms capturing the geometrical error and the error in the complementary condition. The stability and convergence properties of the mixed scheme are studied in numerical experiments. Finally, the application of the a posteriori error estimates within adaptive schemes is discussed. It can be observed that optimal convergence rates of higher-order can be restored for a variety of contact problems using such adaptive schemes.

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ISOGEOMETRIC ANALYSIS USING T-SPLINES

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KEYWORDS: Isogeometric analysis, T-splines, local refinement, fracture

Isogeometric analysis has emerged as an important alternative to traditional engineering design and analysis methodologies. In isogeometric analysis, the smooth geometric basis is used as the basis for analysis. Most of the early developments in isogeometric analysis focused on establishing the behavior of the smooth NURBS basis in analysis. It was demonstrated that smoothness is an important computational advantage over standard finite elements.

While smoothness is an important consideration, NURBS are severely limited by their tensor product construction. Analysis-suitable T-splines are a superior alternative to NURBS. T-splines can model complicated designs as a single, watertight geometry. Additionally, NURBS are T-splines so existing technology based on NURBS extends to T-splines.

In this talk, we review current progress in analysis-suitable T-spline descriptions and their application to isogeometric analysis. Specifically, the underlying formulation, mathematical properties, and local refinement capability will be described. We will then discuss their application to problems in fracture. In this setting it has been shown that local refinement and smoothness offer important advantages over traditional finite element discretizations.

HIERARCHICAL LOCAL REFINEMENT IN ISOGEOMETRIC ANALYSIS

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KEYWORDS: adaptivity; isogeometric analysis; hierarchical B-splines; local refinement

Local refinement is one of the key issues in isogeometric analysis. Since both the geometric representation and the numerical approximation by means of bi- or tri-variate NURBS have a tensor product structure, a quadrilateral topology of the computational mesh for a single patch model in two space dimensions and a hexahedral topology in three dimensions are unavoidable in this framework. As a consequence, truly local refinement is impossible. T-splines have been shown to be able to break up the rigidity of the NURBS-based isogeometric discretization and are currently investigated in various directions [1,2].

As alternative approach, we present in this contribution an adaptive local refinement technique based on extensions of *hierarchical B-splines*. The fundamental idea of our approach is very simple and goes back to work by Kraft [3]. Essential properties such as linear independence and also the preservation of arbitrary smoothness in the corresponding refined Galerkin basis can be guaranteed easily in this way. Furthermore, we use concepts well-established in finite element analysis to fully integrate hierarchical spline spaces into the isogeometric setting, which allows us to make use of a posteriori error estimation. Numerical results taken from [4] illustrate this promising refinement method.

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FEM IN PROFESSIONAL PRACTICE: THE QUESTIONS OF 'WHAT?' AND 'HOW?'

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KEYWORDS: conceptualization, verification, FEA software, mathematical model, extraction, standardization

The present conference is the latest in a series of conferences that started with p-FEM 2000 in St. Louis. These conferences focused on the use of high order methods in computational solid and fluid mechanics. The rationale for viewing high order methods as separate and distinct from low order ones arose from the way the finite element method (FEM) was developed, implemented and applied rather than from the theoretical foundations of FEM. This presentation will address two questions relevant to both professional practice and ongoing research.

The first question is: What should be computed? This question has to be addressed in the process of conceptualization, the end product of which is a mathematical model [1]. Denoting the exact and finite element solutions respectively by u_{EX} and u_{FE}, the goal of computation should be stated as follows: Determine some specific system response quantity $\Phi(u_{FE})$ such that

$$|\Phi(u_{EX}) - \Phi(u_{FE})| \leq \tau|\Phi(u_{EX})| \quad \text{where } \tau \text{ is a prescribed tolerance.}$$

In structural and mechanical engineering practice the usual goal is to predict conditions that will result in the onset and/or propagation of failure. In those cases $\Phi(u_{EX})$ is a function of random variables. The definition of $\Phi(u_{EX})$ typically involves subjective judgment. The mathematical model has to be formulated such that for a fixed set of input data $\Phi(u_{EX})$ exists and is unique, see for example [2]. The suitability of $\Phi(u_{EX})$ for the intended purpose is tested in validation experiments [1].

The second question is: How $\Phi(u_{FE})$ should be computed and how the realized value of τ, denoted by τ_R, can be estimated? A closely related question is: How does one ensure that $\tau_R \leq \tau$? Hierarchic sequences of finite element spaces and properly designed extraction procedures play a very important role in the estimation and control of τ_R.

The professional practice of numerical simulation based on FEM is dominated by software products that make it difficult or impossible to estimate τ_R. This is because model definition and discretization are mixed in the element libraries of these software products and ad-hoc procedures, such as reduced integration and the use of tuning parameters, introduce errors that are outside of the users' control.

New procedures of standardization are being developed that make quality assurance in numerical simulation feasible in professional practice through the utilization of hierarchic models and finite element spaces [3].

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p-FEMs FOR BIOMECHANICAL APPLICATIONS: BONES AND ARTERIES

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KEYWORDS: Femurs, Arteries, Hyperelasticity

The advantages of the *p*-version of the finite element method (FEM) are exploited herein to address the mechanical response of human femurs and arteries.

p-FE models for patient-specific femurs are being constructed automatically from quantitative computed tomography (qCT) scans with inhomogeneous orthotropic linear elastic material assigned to the FE models directly from the CT scan at each integration point [1]. These FE models are being verified and thereafter validated on a cohort of 17 fresh-frozen femurs which were defrosted, qCT-scanned, and thereafter tested in an in-vitro setting [2,3].

We also investigate the complex mechanical response of human arteries by $p-$FEMs, addressing geometrically non-linearity involving hyperelastic anisotropic, nearly-incompressible constitutive models with passive and active parts [4]. In this case we demonstrate the efficiency of *p*-FEMs compared to traditional commercial *h*-FEMs as Abaqus and the locking free properties of the displacement-formulation when applied to nearly incompressible hyperelastic materials under finite deformations.

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Sparsity optimization of H(div)- and H(curl)-conforming hp-finite elements

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KEYWORDS: hp-finite elements, de Rham Complex, sparsity optimization, symbolic summation tools

The main feature of hp-finite element methods are their extremely fast convergence properties with respect to the number of unknowns. But with increasing the polynomial order the density of element matrices as well as the costs of numerical integration gets crucial. On tensor-product elements one can easily overcome these difficulties by constructing a product-based finite element basis exploiting the orthogonality relations of 1d-Legendre-type polynomials. As initially suggested by Dubiner and Karniadakis-Sherwin using the Duffy transformation and Jacobi-type polynomials with adapted weights are the remedy for simplicial elements in case of the scalar function spaces L2(W) and H1(W).

In this talk we are concerned with the vectorvalued function spaces H(div) and H(curl) which occur e.g. in various formulations of fluid mechanics, in mixed formulations of elasticity or the system of Maxwell's equation. A conforming and stable hp-fe discretization first requires continuity conditions over element interfaces as well as global exactness in the sense of de Rham. For reasons of stability and parameter-robust preconditioning we rely on a finite element basis which explicitly includes a basis of the higherorder functions as suggested by Schöberl and Zaglmayr. This technique turns out to be a further key tool to extend the techniques of sparsity optimization to H(div)- and H(curl)- conforming hp-FEM. We discuss the construction principles of the new fe-basis and the analysis of the sparsity pattern of several problems, for which we also use symbolic computation in 3D. We conclude with numerical experiments, which in addition to the proven sparsity, also show tremendously improved condition numbers of the high-order finite element matrices.

Converting Unstructured Quadrilateral/Hexahedral Meshes to T-splines

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KEYWORDS: quadrilateral mesh, hexahedral mesh, T-spline surface, solid T-spline, extra-ordineary node, sharp feature

A crucial work in Reverse Engineering is to construct 3D geometry from scanned imaging data. With the rapid development of 3D scanning data acquisition and automatic mesh generation techniques, the geometries we obtained are always in the form of polygonal meshes. Besides polygonal meshes, spline is another widely used representation of freeform geometry, especially in Computer Aided Design (CAD), Computer Aided Manufacturing (CAM), and Computer Aided Engineering (CAE). Spline representation has many good properties, such as high geometric accuracy and high order continuity. Recently, a spline-based analysis method named isogeometric analysis was developed, which shows great advantages over traditional finite element analysis. To improve the geometric accuracy and continuity, and also to make the model compatible with CAD systems and isogeometric analysis, it is always desirable to convert the polygonal meshes into continuous, high-order spline surfaces.

In this talk, we present a novel method for converting any unstructured quadrilateral or hexahedral mesh to a T-spline surface or solid T-spline. The T-spline definition is generalized based on the rational T-spline basis functions. Based on this generalized definition, our conversion algorithm contains two stages: the topology stage and the geometry stage. In the topology stage, the input quadrilateral or hexahedral mesh is taken as the initial T-mesh, templates are designed for each quadrilateral element type in 2D or for each hexahedral mesh node in 3D in order to get a gap-free T-spline. In the geometry stage, an efficient surface fitting technique is developed to improve the surface accuracy with sharp feature preserved. Finally, a Bézier extraction technique is used to facilitate T-spline based isogeometric analysis.

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Contributed posters

The Finite Cell Method–
Adaptive integration and application to problems of elastoplasticity

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KEYWORDS: Finite Cell Method, elastoplasticity, numerical integration schemes.

The Finite Cell Method (FCM) [1, 2] can be considered as an embedding or fictitious domain method combined with high-order finite elements [3]. In this embedding method, the mesh is not necessarily conforming to the boundaries of the physical domain. Applying the FCM, the boundary is extended to a simple domain which can be discretized easily utilizing a Cartesian grid of cells. The geometry is accounted for during the integration of the stiffness matrices and the accuracy of the approximation is controlled by increasing the polynomial degree of the shape functions of the cells. Thus, the effort of meshing complex domains is replaced by the task to accurately perform the numerical integration of the cells.

The FCM enjoys fast convergence in terms of the degrees of freedom when performing a p-extension. However, the computational cost of the method depends strongly on the integration scheme. Fast and accurate integration of discontinuous functions is still a challenge for all fixed mesh finite element based methods, including the FCM. Several integration schemes will be examined and modifications will be proposed for discontinuous integrands. The adaptive integration schemes will be compared for problems of two- and three-dimensional elasticity. In addition we will also extend the FCM to elastoplastic problems and investigate its performance for this type of nonlinearity.

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A DISCONTINUOUS PETROV-GALERKIN METHOD FOR LINEAR ELASTICITY, CRACOW 2011

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KEYWORDS: DPG, linear elasticity, wave propagation, pollution error

In this research, we present Discontinuous Petrov-Galerkin (DPG) finite element methods for static and time-harmonic linear elasticity. For both cases, we define optimal test functions which are shown to deliver the best approximation error if an optimal global test norm is used. To make the methods practical, we use a localizable test norm. In the static case, it can be shown that this norm is equivalent to the global optimal norm. The majority of this proof is the verification that the inf-sup condition holds for the DPG formulation using the localizable test space norm. From DPG theory, this proves our method is quasi-optimal with a constant independent of the mesh. We can then use results from approximation theory to show h and p convergence for the static case.

Using the localizable test space norm, we have implemented the static formulation and show h and p convergence of the method at optimal rates. Additionally, the DPG framework provides an a posteriori error estimator determined by solving local auxiliary variational problems. We use this estimator as the basis for various greedy adaptive schemes. We test our adaptive algorithm using a manufactured smooth solution as well as a singular solution L-shape domain problem and observe adaptive h and hp convergence.

The method can also be extended to time-harmonic elastic wave propagation problems. A key feature of the DPG method is the reduction of pollution error, and can therefore be used to solve problems with a large number of wavelengths. Due to this fact, we will present numerical results for time-harmonic elastic wave problems with high wave numbers.

The principal contributions of this research are proving p convergence for the dual-mixed static elasticity system, particularly without the need for a discrete exact sequence or commuting diagram, as well as a practical adaptive 2D time-harmonic elasticity code with a posteriori error estimation which can be used for high wave number problems. In this poster, we will present an overview of the theoretical DPG framework, the specific formulation for both static and time-harmonic elasticity, and the numerical results for both cases.

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A Multiscale hp-FEM for 2D Photonic Crystal Bands

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KEYWORDS: Generalised FEM, Helmholtz equation, Periodic Media.

A Multiscale Finite Element Method (MSFEM) for wave propagation in locally periodic media, e.g., Photonic Crystals (PhC), will be presented [2]. We consider wave propagation at wavelengths of the size of the local periodicity. In this case homogenisation techniques are not applicable. The MSFEM is g-FEM variant [1, 3, 4], which uses two-scale basis functions inside the PhC. As micro functions we use Bloch modes, which are computed as FEM solutions for a fully periodic PhC [5]. The macro functions are piecewise polynomials of degree p^{mac} which are supported over many periods of the crystal. We will numerically show that such a multiscale basis is very efficient as only a constant number of these functions are needed to simulate arbitrary large, finite PhCs with a constant L_2-error. In contrast, for standard discretisation schemes like h-, p- or hp-FEM more and more basis functions are required when the number of scatterers increases inside the computational domain. We will explain how to use this multiscale basis to construct a conforming FEM which is coupled to a discretisation of the exterior domain. In particular, we will show how to numerically integrate the highly oscillatory two-scale functions with constant computational effort. We will verify the properties of the MSFEM by numerical experiments for PhC bands (see Fig. 1), an infinite band of locally-periodic dielectric scatterers in 2D.

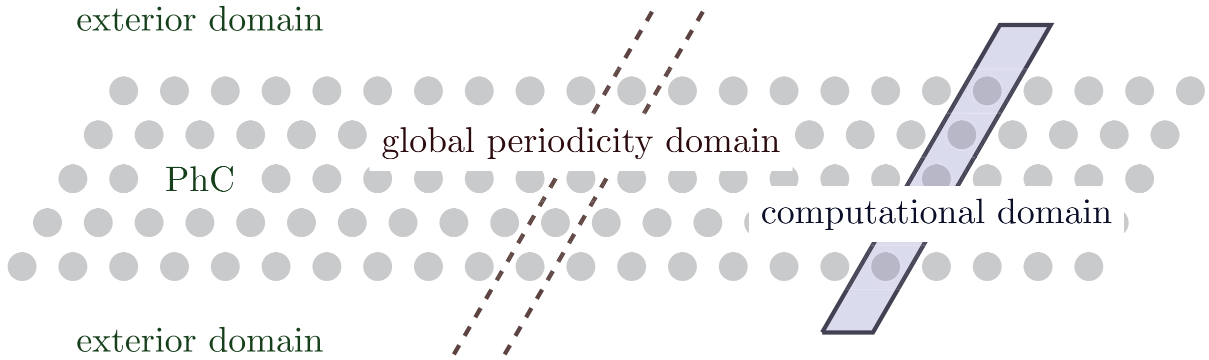

Figure 1: An example Photonic Crystal band

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Towards an hp-Adative Refinement Strategy for Maxwell's Equations

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KEYWORDS: hp-adaptive, a posteriori error estimate, Maxwell's equations, finite element method

The finite element method provides an efficient framework for the numerical solution of partial differential equations. Its performance can be improved by mesh refinement (h-refinement) or the use of higher order ansatz spaces (p-refinement). A combination of both (hp-refinement) can lead to exponential convergence rates of the computed solution.

Nowadays adaptive refinement of the computational domain is a widely used feature to obtain an accurate numerical solution of the partial differential equation with as less computational work as possible. Therefore one needs to decide, where the approximation error of the numerical solution is relatively large and, thus, refinement should take place. Since the analytic solution is usually not known, one has to estimate the approximation error in terms of the numerical solution to be able to decide which areas of the computational domain have to be refined further.

In recent years a broad interest in the numerical solution of Maxwell's equations has come up, because this system of equations appears in a lot of nano-scaled processes due to the presence of an electromagnetic field. Solving these equations numerically usually requires a lot of computational work and a fully automatic hp-adaptive refinement strategy can reduce the amount of work, which is related to solving the stationary problems, significantly. We present an hp-efficient residual-based a posteriori error estimator for Maxwell's equations in the electric field formulation, which gives a reliable and robust estimation of the true energy error. The performance of our error estimator is compared to other, well-known, a posteriori error estimators for Maxwell's equations and its application in h- and p-adaptive refinement strategies is shown. This hp-efficient a posteriori error estimator is a first step towards an hp-adative refinement strategy, which produces a sequence of conforming finite element spaces such that the numerical solution of the partial differential equation converges with an exponential rate to the analytic solution of the problem.

OVERLAPPING ADDITIVE SCHWARZ METHODS FOR ISOGEOMETRIC ANALYSIS

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KEYWORDS: Domain Decomposition methods, Overlapping Schwarz, scalable preconditioners, Isogeometric Analysis, finite elements, NURBS

Isogeometric Analysis is a non-standard numerical method for partial differential equations (PDEs), which was introduced by T. J. R. Hughes in [1]. In the isogeometric framework, the ultimate goal is to adopt the geometry description from a Computer Aided Design (CAD) parametrization, and use it for the analysis, that is, within the PDE solver. Non-uniform rational B-splines (NURBS) are a standard in CAD community mainly because they are extremely convenient of the representation of free-form surfaces and there are very efficient algorithms to evaluate them, to refine and derefine them. In IGA, those same basis functions (that represent the CAD geometry) are also used as the basis for the discrete solution space of PDEs, thus following an isoparametric paradigm. IGA methodologies have been studied and applied in fields as diverse as fluid dynamics, structural mechanics and electromagnetics.

Domain decomposition methods are a major area of recent research in numerical analysis for PDEs. They provide robust, parallel and scalable preconditioned iterative methods for the large linear systems arising in discretizaton of the continuous problems.

In this poster, we propose overlapping additive Schwarz methods for elliptic problems in Isogeometric Analysis. We construct additive Schwarz preconditioners both in the parametric space and in the physical space and also prove that our proposed methods in multi-dimensions are scalable. Moreover, we present a set of numerical experiments, including the case with discontinuous coefficients, which is in complete accordance with the theoretical developments.

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THE COST OF CONTINUITY: A STUDY OF THE PERFORMANCE OF ISOGEOMETRIC FINITE ELEMENTS USING DIRECT SOLVERS

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KEYWORDS: direct solver, isogeometric analysis *k*-refinement

In this poster we will present findings from a study conducted to improve our understanding of the cost associated with solving linear systems resulting from isogeometric analysis–where each basis possesses higher order continuity than conventional finite element analysis.

Isogeometric analysis has received a lot of attention in recent years, originally motivated by the desire to find a method for solving partial differential equations which would simplify, if not eliminate, the problem of converting geometric discretizations in the engineering design process. Tangential to the benefits of geometry/analysis unification, the method is also well suited for solving nonlinear and higher-order PDE's due to its higher-order continuity. A wide variety of application areas have taken advantage of the strengths of isogeometric analysis. These applications include structural vibrations, fluid-structure interaction, patient-specific arterial blood flow, complex fluid flow and turbulence, shape and topology optimization, phase field models via the Cahn-Hilliard equation, cavitation modeling, and shell analysis

This initial study focuses on the use of direct solvers, specifically MUMPS. Direct solvers were chosen for this work for several reasons. First, while implementations of direct solvers vary, the algorithm is still LU-factorization and therefore the trends apply for a wide variety of direct solvers. Also, direct solvers are important for problems with multiple right-hand sides, such as goal-oriented adaptivity or inverse problems.

The results show that degrees of freedom in systems whose basis possesses higher degrees of continuity can cost 2-3 orders of magnitude more time and memory to solve. This is due to the matrix becoming more dense as continuity approaches its maximum, at C^{p-1}. This finding adds a dimension of complexity in the choice of refinement scheme. While k-refinements are economic in terms of degrees of freedom, each degree of freedom costs more to solve in a direct solver.

A RECIPE: HOW TO CONSTRUCT A ROBUST DPG METHOD FOR THE CONFUSION PROBLEM (AND ANY LINEAR PROBLEM AS WELL)

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The main promise of the *Discontinuous Petrov-Galerkin Method with Optimal Test Functions* introduced by Demkowicz and Gopalakrishnan in [1] is that the discrete problem automatically inherits stability from the continuous one. This fundamental property, true for any linear problem described with a system of first order PDEs, is accomplished by computing on the fly, element by element, optimal test functions. The method can be interpreted as a special least squares method for the operator equation:

$$Bu = l, \quad B : U \to V', \, l \in V'$$

Here U and V are two Hilbert spaces and V' denotes the dual of V. Given a finite-dimensional trial subspace $U_h \subset U$, the approximate solution u_h is obtained by minimizing the residual in the dual space,

$$\|Bu_h - l\|^2_{V'} \to \min$$

Recalling that Riesz operator $R_V : V \to V'$ is an isometry, we can replace the dual norm in the minimization problem with the test norm,

$$\|R^{-1}(Bu_h - l)\|^2_V \to \min$$

The minimization problem is then equivalent to the variational problem,

$$(R^{-1}(Bu_h - l), R^{-1}B\delta u_h)_V = 0, \quad \forall \delta u_h \in U_h \tag{1}$$

or, equivalently, a Petrov-Galerkin discretization of the original problem

$$< Bu_h, v_h > = < l, v_h >, \quad \forall v_h \tag{2}$$

with optimal test functions $v_h = R^{-1}B\delta u_h$ that are obtained by solving the auxiliary variational problem:

$$(v_h, \delta v)_V = < Bu_h, \delta v >, \quad \forall \delta v \in V \tag{3}$$

The first critical point for the practicality of the approach is the use of *discontinuous test functions* which enables the elementwise inversion of the Riesz operator. This implies the use of ultra-weak variational formulation resulting in a *hybrid* method: on top of standard (field) variables, we need to solve for *traces* and *fluxes* that live on interelement boundaries (the *mesh skeleton*). Obviously, the method depends upon the choice of the test norm. Different test norms lead to different *mapping* (continuity) properties for the original problem that are then inherited by the DPG method. The problem of choosing the right test norm becomes critical for *singular perturbation problems* where we strive not only for the stability of the discretization but also for *robustness*. i.e. a stability that is uniform with respect to the perturbation parameter. We shall use the convection-dominated diffusion problem to demonstrate how one systematically introduce different test norms that guarantees the robustness. This brings us to the second critical point of the approach. In practice, the optimal test functions are determined only approximately by using a Bubnov-Galerkin approximation to problem (3) in an *enriched space*, a standard practice in *implicit* a-posteriori error estimation. Some obvious choices of test norms guaranteeing robustness lead to optimal test functions that exhibit boundary layers (depending upon the perturbation parameter) that are difficult to resolve. We will show how to construct a test norm that does not have this property and leads to optimal test functions that can be easily resolved using the enriched spaces. The work generalizes the 1D construction reported in [2].

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BOUNDARY CONDITIONS AND MULTI-PATCH CONNECTIONS IN ISOGEOMETRIC ANALYSIS

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KEYWORDS: Isogeometric Analysis, Boundary Conditions, NURBS Multi-Patch Connection

The aim of our research project is to examine the applicability of isogeometric Finite Element Analysis for daily use in structural analysis. For this purpose surface elements are used as those prevail in most engineering applications. Regarding the basics of Isogeometric Analysis and element formulations see e.g. [1] and the references therein. Imposing boundary conditions, handling of different kinds of loads and connection of neighboring domains are for practical use of outstanding importance. This contribution focuses on the weak imposition of Dirichlet boundary conditions and connection of multiple patches and compares the results to strong imposition.

The studies are conducted in a self-developed MATLAB isogeometric framework. This allows for a maximum of control and flexibility. For the sake of simplicity plane shell elements are used, as the focus is laid on boundary and transition conditions. The results are easily transferable for more complex elements. In the case of weak imposition the treatment of boundary and transition conditions is quite alike and realized with the Lagrange Multiplier method and with the Perturbed Lagrangian method as described in [2]. For both the weak form of equilibrium is enriched with additional terms to constrain deformations. Consistent tangent matrices and residua are derived. The implementation is realized with connection elements on NURBS curves. The results of standard benchmarks are compared to computations with strong imposition, i.e. shared degrees of freedom for the connection of patches and elimination of degrees of freedom for Dirichlet boundary conditions.

Computations using the Lagrange Multiplier method show exactly the same mesh convergence as corresponding systems with strong imposition. Convergence rates for geometries with hanging nodes are compared to convergence rates of conforming meshes. The parameterization of the connection elements and the selection of active Lagrange degrees of freedom have to fulfill certain requirements to generate a well-conditioned system of equations. The results for the Perturbed Lagrangian method depend on the choice of the penalty parameter α. For an appropriate α the mesh convergence is indistinguishable from the Lagrange Multiplier method and quadratic convergence in nonlinear computations can be observed. In nonlinear computations it is possible to adapt α with the help of displacement norms.

The results clearly show that both presented methods are convenient to impose boundary conditions and connect patches in linear and nonlinear computations. The Lagrange Multiplier method has two major drawbacks. The structure of existing FEA-codes has to be modified for the additional unknowns. Furthermore the indefiniteness of the stiffness matrix due to zeros on the diagonal requires specialized solvers. The Perturbed Lagrangian method does not require special solvers nor a changed structure of the FEA-code. The only drawback is the need to determine the penalty parameter. Despite these disadvantages the presented weak connection methods seem to be the only feasible way for the connection of non-conforming meshed patches, which is a prerequisite for local refinement. The tensor product character of NURBS surfaces leads to quadrilateral meshes. A direct connection of patches would require all control points of the considered edge of both patches to coincide. It is possible to create triangular patches at the interface between patches, but this would infringe the isogeometric idea as the geometry had to be altered for analysis.

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A BOUNDARY CONFORMAL APPROACH FOR A HIGHER ORDER DISCONTINUOUS GALERKIN FINITE ELEMENT METHOD

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KEYWORDS: Discontinuous Galerkin, boundary conformal approximation, hybridization

A boundary conformal approach for solving three dimensional electro-quasistatic problems with a high order Discontinuous Galerkin (DG) method on Cartesian grids is proposed. The material boundary subdivides the Cartesian grid cells into sub-cells which are associated with (at least) two different sets of material parameters. We will refer to them as cut-cells. The challenge consists in deriving an appropriate numerical approximation within these cells. Since no general set of basis functions satisfying continuity conditions can be defined for an arbitrarily shaped cut-cell, the standard Continuous Galerkin Finite Element Method (CG-FEM) formulation cannot be applied. Instead, we propose a formulation based on the high order DG method as introduced in [1]. The cut-cell approach can be naturally embedded within the DG framework which does not impose conformity conditions on the approximation spaces. In the present implementation, for each sub-cell, a set of independent high-order hierarchical basis functions proposed in [2] is specified. The basis functions within the sub-cells are chosen to be identical with those in the parent Cartesian cell.
The numerical evaluation of the DG integrals, however, needs an appropriate description for the cut-cell geometry. For this purpose, the Open CASCADE geometry kernel [3] is used. It enables a geometrical representation of the cut-cells based on parameterized Bezier and B-Spline surfaces. Furthermore, a particular numerical quadrature technique is applied which allows for an accurate integration of the finite element operators taking into account the exact geometry of the cut-cells.

Depending on the problem geometry cut-cells with small sub-cell volumes may occur which effect the condition number and cause solver convergence problems. In order to handle this problem, a merging method is introduced which merges small sub-cells with the neighboring Cartesian grid cell that has the largest shared surface. Consequently, the condition number of the system decreases significantly.

Finally, a hybridization method is proposed which sets up a relation between the CG-FEM and the DG solution space. The DG degrees of freedoms in normal Cartesian grid cells are reduced to CG-FEM degrees of freedoms by the topological projection operators introduced in [4]. This approach reduces the degrees of freedom in homogenous domains substantially and makes the method more efficient.

Numerical examples are presented which demonstrate the optimal convergence rate, $P+1$, where P is the highest degree of polynomials of the method for any approximation order and problem geometry.
The strength of the boundary conformal approach consists in its capability to obtain high order accuracy solutions on arbitrary domains with trivial meshes. The discrete problem formulation is simple and straightforward to implement.

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AUTOMATIC *HP*-ADAPTIVITY FOR THREE DIMENSIONAL ELECTROMAGNETIC PROBLEMS. APPLICATION TO WAVEGUIDE PROBLEMS

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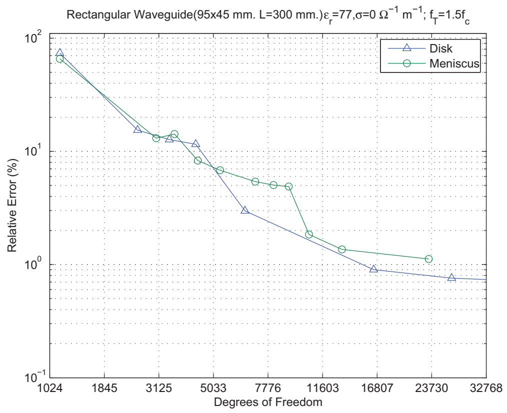
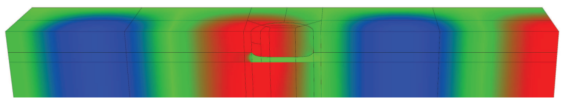
KEYWORDS: *hp*-adaptivity, electromagnetics, three dimensional analysis, waveguide problems

In this contribution, an automatic *hp*-adaptivity for three dimensional (3D) closed domain electrodynamic problems is presented. It is based on the work on *hp*-adaptivity leaded by Prof. Demkowicz of the University of Texas at Austin, [1]. The three dimensional *hp*-strategy supports anisotropic refinements on irregular meshes with hanging nodes, and isoparametric elements. Specifically, it makes use of hexaedral elements and $\mathbf{H}(\text{curl})$ higher order discretizations. In the poster presentation, results of its application to the double-curl vector formulation in the context of waveguiding problems will be shown. Specifically, the results will include the characterization (in terms of its scattering parameters) of several rectangular waveguide discontinuities used in microwave engineering. Exponential convergence of the error in the energy norm will be shown.. In the following figures, some results of the analysis of the electromagnetic field distribution in a petri dish with a cell culture inside a rectangular waveguide are presented. The effect of the meniscus, which has a sensible influence in the electromagnetic field distribution, is observed.



(a) Error convergence



(b) Electric field inside the waveguide. Meniscus effect.

Figure 1: Petri dish problem

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ON REGULARIZATION AIDED HO MULTIPOINT SOLUTION APPROACH

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KEYWORDS: higher order approximation, multipoint approach, regularization

The higher order approximation multipoint technique in the meshless FDM is considered. It is based on arbitrary irregular meshes, MWLS approximation [5] and the global, local or global-local formulations of b.v. problems [4].

In the multipoint formulation, following the original Collatz [1] multipoint concept, the meshless FDM (MFDM) difference operator Lu is obtained by the Taylor series expansion of unknown function u including higher order derivatives and using additional degrees of freedom at nodes. For this purpose one may apply e.g. combination of the right hand side values f_i of the considered differential equation (which are known values) at any node of each MFD star using arbitrarily distributed clouds of nodes:

$$\mathcal{L}u_i \approx Lu_i \equiv \sum_j c_{ij} u_j = \sum_j \alpha_{ij} f_j \quad \Rightarrow \quad Lu_i = Mf_i \ .$$

It is the basic formula for the multipoint specific formulation. Here, j – number of a node in a selected FD star, Mf_i – a combination of the equation right hand side values, f_i – may present value of the whole operator $\mathcal{L}u_i$ or a combination with its derivatives. In the general multipoint method, a specific derivative $u_i^{(k)}$ (a part of the whole operator $\mathcal{L}u_i$ only) is used as additional d.o.f. instead of the right hand side of the given differential equation

$$\sum_j c_{ij} u_j = \sum_j \alpha_{ij} u_j^{(k)} \ .$$

Application of the specific approach is mainly restricted to the linear b.v. problems. The general formulation is more complex but it may be used for all types of b.v. problems (e.g. for non-linear ones).

Unfortunately, one may encounter ill-conditioning phenomenon of the system of MFDM algebraic equations, when the general multipoint formulation is considered for two-dimensional problems [3].

Such situation happens e.g. when we evaluate higher order derivatives with the respect to y by using nodal values of u and u_x as d.o.f. One may avoid such situation assuming such d.o.f. of the MFD star, that allow for evaluation of all partial derivatives up to p-th order. Each time the critical question is the ability of inversion of the basic matrix involved in generation by MWLS approximation of the MFD formulas. In the case of PDE have been proposed and tested several different general versions of multipoint algorithm therefore. However, the problem of their effectiveness is the main issue now.

The simplest solution in this situation could be using a regularization term [2] in the weighted error functional built for the MWLS approximation. The preliminary tests of regularization used in multipoint approach analysis were carried out. Results of tests and their comparison with those obtained for the previous developed general multipoint versions are encouraging. The method of regularization is under current investigation.

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Adaptive refinement in isogeometric analysis using LRB-splines

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KEYWORDS: isogeometric analysis, local refinement, LRB-splines

Isogeometric analysis as introduced by Hughes et.al [2] in 2005 is quickly growing into a mature research field. Using NURBS as basis functions in a finite element method has shown to have some remarkable advantages, not only for their superior properties of representing complex geometries, but also for their numerical properties. NURBS is however limited to tensor product configuration, making local refinement inherently a hard problem to solve. Over the years, several techniques have been employed to solve this. T-splines as introduced by Sederberg et.al [3] in 2003 was one of the recent contributions to this research field and have seen a lot of attention from computer aided engineering (CAE). T-splines was, as the years indicate, developed prior to isogeometric analysis, and its use in finite element problems have not been entirely straightforward. Later years have seen the birth of another approach in the Locally Refined B-splines (LRB-splines) by Dokken et.al [1] in 2011. We will investigate the use of LRB-splines as a basis for doing finite element computation, and also their ability to do local refinement.

Both T-splines and LRB-splines refinement schemes are in general propagating, but this is due to fundamentally different reasons. This means that typically when inserting continuity reduction lines in a mesh, the methods will require more lines than you request. This effect can be diminished by certain techniques, and we will illustrate how to achieve ”perfect” local refinement in the sense that knot lines does not propagate at all.

LRB-splines open up several design parameters when creating refinement schemes, namely:

- which basis functions with support on the element to refine
- the location of the element split
- the multiplicity of the inserted knot lines
- (degree elevation)

We will take an in depth look at what effects different refinement techniques have on our solution algorithms when used as a basis for finite element computations.

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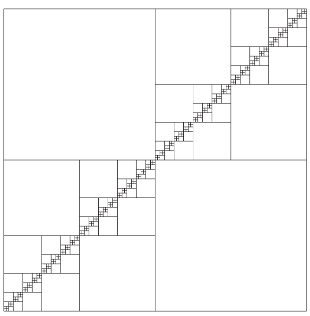

Figure 1: Diagonal ”perfect” refinement

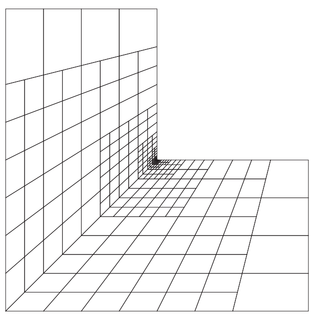

Figure 2: L-shape singularity

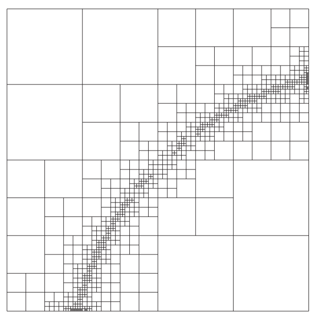

Figure 3: Front solution

CONSTRAINED APPROXIMATION IN hp-FEM – NON-MATCHING REFINEMENTS AND MULTI-LEVEL HANGING NODES

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KEYWORDS: constrained approximation, hanging nodes, hp-fem

This poster presents concepts for the implementation of constrained approximation for conforming hp-adaptive finite element schemes with non-matching, unsymmetric refinements and multi-level hanging nodes.

In order to apply conforming finite element schemes to meshes with hanging nodes, certain measures need to be taken to ensure continuity. Complex refinement patterns possibly created by additional local refinements can be avoided by constraining the associated degrees of freedom via the well-acknowledged approach of constrained approximation, cf. [1, 2, 3]. For this, a representation of shape functions in terms of transformed shape functions, given by so-called constraint coefficients, is necessary, cf. [4, 5]. The refinement of mesh elements containing hanging nodes may lead to multi-level hanging nodes which significantly complicates the derivation and implementation of constraint coefficients if no further restrictions such as 1-irregularity of the mesh or symmetry of the refinements are imposed, cf. [1, 6, 7]. Additional difficulties arise when adjacent elements are refined using unsymmetric, non-matching refinements, i.e. their common edge is refined in two different ways resulting in four partly overlapping constrained edges.

We present an hp-FEM implementation which handles arbitrary non-matching refinements with multi-level hanging nodes. Using recursive formulas for constraint coefficients, the coupling of constrained degrees of freedom to their constraining degrees of freedom is computed using a globally defined connectivity matrix. This approach entirely avoids the need for mesh regularization allowing for an almost unrestricted choice of refinement schemes.

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A TWO-FIELD DUAL-MIXED *hp* FINITE ELEMENT MODEL FOR CYLINDRICAL SHELLS

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KEYWORDS: two-field, dual-mixed, dimensional reduction, cylindrical shell, *hp* FEM

Improving the efficiency and accuracy of the computed stresses, the variable of primary interest in many engineering applications, is still a demanding task. The application of the the two-field dual-mixed variational principle of Fraeijs de Veubeke is demonstrated for plane and plate problems in [1,2] and for cylindrical shells in [4].

The derivation of our cylindrical shell model is based on Fraeijs de Veubeke's variational principle [3] with functional

$$\mathcal{F}(\sigma^{pq}, \phi_{pq}) = \int_V \left[W_c(\sigma^{pq}) + \phi_{pq}\sigma^{pq} \right] \mathrm{d}V - \int_{S_u} \tilde{u}_p \sigma^{pq} n_q \,\mathrm{d}S, \qquad (1)$$

where σ^{pq} is the not *a priori* symmetric stress tensor, ϕ_{pq} is the skew-symmetric (infinitesimal) rotation tensor, V denotes the volume of the body bounded by surface $S = S_u \cup S_p$ ($S_u \cap S_p = \emptyset$) and $\tilde{u}_p$ is the prescribed displacement vector on the surface part S_u. The complementary strain energy is given by $W_c(\sigma^{pq}) = \frac{1}{2}\sigma^{k\ell} C^{-1}_{k\ell pq}\sigma^{pq}$, where $C^{-1}_{k\ell pq}$ is the fourth-order elastic compliance tensor. The subsidiary conditions to functional (1) are the translational equilibrium equations $\sigma^{k\ell}_{\ ;\ell} + b^k = 0$ and the stress boundary conditions $\sigma^{k\ell} n_\ell = \tilde{p}^k$ on S_p, where b^k is the density vector of the body forces and $\tilde{p}^k$ is the prescribed surface traction vector on the surface S_p with outward unit normal n_ℓ. The skew-symmetry condition for the rotation tensor, $\phi_{pq} + \phi_{qp} = 0$, is an additional subsidiary condition. The displacement boundary conditions on the surface S_u are imposed weakly. The tensor of first-order stress functions $\Psi^k_{.p}$ is applied to satisfy translational equilibrium equations: the stress tensor is expressed by equation

$$\sigma^{k\ell} = \varepsilon^{\ell pq} \Psi^k_{.q;p} + \hat{\sigma}^{k\ell}, \qquad (2)$$

where $\varepsilon^{\ell pq}$ is the third-order permutation tensor and $\hat{\sigma}^{k\ell}_{\ ;\ell} = b^k$.

To derive a dimensionally reduced shell model, all the variables are expanded into power series with respect to the thickness coordinate. The approximation of the stresses through the thickness follows from the structure of the translational equilibrium equations (written in terms of the expanded stresses). A shell model derived in this way makes the application of the classical kinematical hypotheses (regarding the deformation of the normal to the shell middle surface) unnecessary. The numerical performance of a new dual-mixed *hp*-version shell finite element is investigated and presented for cylindrical shells. It will be shown that this element gives reliable and accurate stress computations not only for higher-order *p*-, but also for low-order *h*-type approximations.

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TO MESH OR NOT TO MESH
that is the question

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KEYWORDS: Isogeometric Analysis, Finite Cell Method, p-FEM, mesh generation

This contribution is intended to encourage a discussion on the geometric description of computational models, the necessity to generate conforming meshes and how to avoid them.

Mesh generation for structural analysis is still an important issue. As pointed out by Cottrell et. al [1], 80% of the total time spent for the analysis is devoted to the creation of a suitable geometry and the generation of a computational mesh. Only 20% of the total time are actually spent for the analysis itself. This ratio has been even more unfavorable for high order methods, especially for complicated geometries.

To ease this issue, the authors are involved in the development of a mesh generator. It is currently capable of generating hexahedral, high order meshes for thin walled, curved geometries. In this contribution, we will present some geometrically elaborate examples and demonstrate the techniques with which corresponding, conforming meshes were created.

On the other hand, two different but combinable strategies exist to avoid the generation of meshes all together:

In *Isogeometric Analysis* [1] the aim is to compute directly on a geometry represented by NURBS by utilizing the same higher order shape functions in the analysis. One, thus, avoids having to generate a conforming mesh by drawing it. This leads to relatively coarse meshes but finer ones may be generated by a regular subdivision if the solution demands so.

In the *Finite Cell Method* [2], even very complex geometries of the physical domain can be taken into account at the integration point level. For this purpose, high order shape functions are spanned by a Cartesian grid which embeds the structure to be computed. The geometrical description in the finite cell method does not necessarily need to stem from an analytical formulation. Instead, BREP models of any kind suffice. Equally well, any form of implicit geometric descriptions such as voxel models or models based on space trees may be used without difficulty. This adds flexibility, especially in three dimensions where conform mesh generation can be a challenging task. Examples demonstrating the features of this approach will be presented.

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B-SPLINE FINITE ELEMENT RESPONSE OF ELASTIC BAR UNDER SHOCK LOADING

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KEYWORDS: elastic wave propagation, finite element method, isogeometric analysis, B-spline

The spatial discretization of elastic continuum by *finite element method* (FEM) [1] introduces dispersion errors to numerical solutions of stress wave propagation. When these propagating phenomena are modeled by FEM the speed of a single harmonic wave depends on its frequency and thus a wave packet is distorted. Moreover, the oscillations near the sharp wavefront in FE solution (called Gibb's effect) appears. For higher order Lagrangian finite elements there are the optical modes in the spectrum resulting in spurious oscillations of stress and velocity distributions near the theoretical sharp wavefront [2]. Furthermore, the high mode behaviour of classical finite elements is divergent with order of approximation of a field of displacements.

The modern approach to FEM presents *isogeometric analysis* (IGA) [3]. This numerical method uses spline basic functions as shape functions. IGA approach shows very good frequency and dispersion properties [3] due to the smooth approximation of a displacement field. Therefore, the high mode behaviour of B-spline FEM is convergent with polynomial order of approximation [3]. In this contribution, B-spline FEM is tested in one-dimensional axial propagation of elastic wave in a bar under force loading by Heaviside step function (Fig. 1). The response of the elastic bar is computed numerically by modal superposition method with respect to all eigenfrequencies and by Newmark method (the average acceleration method) [1]. In Fig. 1, the stress σ along this bar discretized by linear ($p = 1$) and cubic ($p = 3$) B-spline and Lagrangian finite elements is depicted for time $t = 0.5L/c_0$. For the B-spline approach, uniformly-spaced and Greville control points are employed. Time step Δt for Newmark method is chosen by Courant number $Co = \Delta t c_0/H_{min} = 0.25$, where H_{min} is minimal distance between control points or nodes. For all models, number of degrees of freedom is 201.

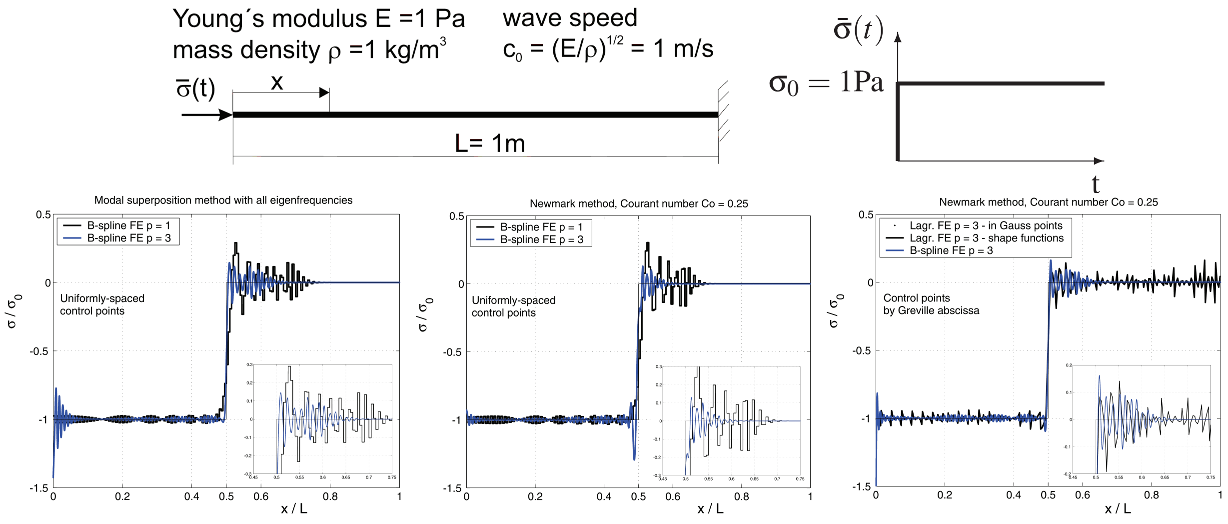

Figure 1: Response of elastic bar under shock loading for different types of discretization and time integration.

In the numerical test, the oscillations near the sharp wavefront for B-spline based FEM are smaller than for classical FEM due to the variation diminishing property. IGA concept has a potential to be efficiently emloyed in high performance and accurate FE analysis of linear and nonlinear wave propagation problems.

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POWERXCELL IMPLEMENTATION OF NUMERICAL INTEGRATION FOR HIGHER ORDER ELEMENTS

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KEYWORDS: numerical integration, PowerXCell processor, higher order elements

In our work we investigate opportunities for parallel implementation of numerical integration algorithm on PowerXCell 8i processors [1]. These processors are a transitional form between the classical architecture of processors and the architecture of GPUs. With fast local memory that can be managed at a user level and eight vector cores it is possible to achieve high performance for many complex scientific calculations.

In the paper we consider the algorithm of numerical integration for higher order finite elements [2]. We design and implement several parallelization strategies for higher order prismatic elements and test their performance for different situations. The different strategies are based on different loops in the integration algorithm for which parallelization is applied. Parallelization strategy I is based on the loop over elements, strategy II is based on the loop over integration points and strategy III is based on parallelization of the outer loop over basis functions [3].

The parallel algorithms are tested for discontinuous Galerkin finite elements with different degrees of approximating polynomials. The results show the potential for speed-up and indicate critical places in the algorithm and some peculiarities of the PowerXCell 8i processor and its programming model.

As a result of our investigations and tests we received complex analysis of performance obtained from using heterogenous processing units with specialized vector coprocessors. This results can be used as a guideline for choosing the proper parallelization strategies for different orders of finite elements in differents problems.

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DPG METHOD BASED ON THE OPTIMAL TEST SPACE NORM FOR STEADY TRANSPORT PROBLEMS

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KEYWORDS: convection-diffusion, discontinuous Petrov-Galerkin,finite element method

The success of the traditional Ritz-Bubnov-Galerkin finite element method in most structural problems is based on the so called best approximation property. This means that the difference between the finite element solution and the exact solution becomes minimized with respect to certain norm, often called as the energy norm. The property follows largely from the symmetry of the stiffness matrices that the method produces.

Numerical problems arise when the best approximation property (or the energy norm in the first place) is lost for some reason. This happens, for instance, when the standard Galerkin method is applied to convective transport problems. In these problems, the system matrix associated to convection is not symmetric and numerical solutions tend to show spurious, non-physical oscillations unless the finite element mesh is heavily refined.

This work concerns the finite element analysis of convection dominated flow problems within the recently developed Discontinuous Petrov-Galerkin (DPG) variational framework, see [1]. We demonstrate how test function spaces that guarantee numerical stability can be computed automatically with respect to the so called optimal test space norm. This should make the DPG method not only stable but also robust, that is, uniformly stable with respect to the Péclet number in the current application, cf. [2,3]. We employ discontinuous piecewise Bernstein polynomials as trial functions and construct a subgrid discretization that accounts for the singular perturbation character of the problem to resolve the corresponding optimal test functions. We also show that a smooth B-spline basis has certain computational advantages in the subgrid discretization.

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STREAMLINE UPWIND PETROV DISCONTINUOUS GALERKIN (SUPDG) METHOD FOR SCALAR AND SYSTEM CONSERVATION LAWS

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KEYWORDS: discontinuous Galerkin, SUPG method, strong-stability-preserving Runge-Kutta

The Runge-Kutta Discontinuous Galerkin (RK-DG) methods are ideally suited for high-order approximation [1]. The RK-DG method is stable when applied to linear hyperbolic problems but when the problem is nonlinear spurious oscillations occur near strong shocks or steep gradients. Then the usual approach to calculations of numerical fluxes is very often not sufficient to stabilize the solution for large high-order elements. Various attempts to use slope limiters with a shock detectors to overcome this problem were proposed recently. When DG method is limited most methods reduce the solution to first-order accuracy and much of the advantage of high-order methods is lost [2].

In the present contribution the stream upwind Petrov Galerkin approach, which includes the perturbation of the flux at the boundaries is combined with the nodal discontinuous Galerkin method [3]. The combination is accomplished after discretization in space by using double integrations by parts of the governing equations. This distinguishes the methods from other approaches which incorporate artificial diffusion directly into problem formulation. The advantage of this type discretization relies on increased locality in data dependencies, which leads to more accurate solution representation. The method does not use slope limiters or shock capture terms. The flux derivatives are converted into Jacobian matrices. The form of equations is similar to flux reconstruction (FR) approach [4] and lifting collocation penalty formulation, which is extension of FR to unstuctured meshes [5]. These approaches however solve the conservation laws in the differential form instead of the integral form.

The present DG approximation is accomplished by using polynomial shape functions of high-orders while keeping the stencil local. The integrations are performed using Gaussian integration of moments. The time discretization is achieved using selection of explicit strong stability preserving Runge-Kutta methods. The new schemes have been verified on several benchmark computational test cases (Burger and Euler equations for compressible flow) for discontinuous and continuous initial valued problems. The stability limit has been established using Von Neumann stability analysis. The technique ability to capture the discontinuous shock waves in hyperbolic flow problems is demonstrated using various examples.

The examples demonstrate that the present scheme can work with higher CFL number than the classical DG method. The higher CPU time disadvantaged DG methods when compared with other higher order methods such as spectral volumes due to restrictive nature of an approximate CFL condition for linear stability of RK-DG method [1]. The higher CFL number feature of the present approach is especially important when the order of polynomials spatial discretization increases. The method is highly compact and can easily handle adaptivity strategies allowing to use different polynomials between elements. The simulation can be partitioned into independent operations using modern graphics processing units (GPU).

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Numerical integration on GPUs for higher order finite elements

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Numerical integration forms one of necessary steps in finite element calculations. The terms from a weak statement of a problem are integrated to form entries in the global stiffness matrix. Then the system of linear equations, with the stiffness matrix as the system matrix, is usually solved leading to the approximate solution. On one hand, numerical integration is an embarrassingly parallel algorithm - integrals over the computational domain are sums of integrals over individual elements. Calculations for different elements are independent and can be done in parallel. Moreover, for many problems, like e.g. linear problems with constant coefficients and linear approximation, numerical integration can be simplified and some precomputed quantities used in codes. There are however problems and approximations - e.g. non-linear problems, problems in domains with curved boundaries, higher order or hp-adaptive approximations - where numerical integration has to be performed step by step. These are usually also the cases in which numerical integration forms a substantial part of finite element calculations - both in terms of required CPU time and RAM memory. We concentrate on such cases, using model problems of standard continuous FEM for linear elasticity and discontinuous Galerkin FEM for scalar elliptic equation, both with higher order approximation.

We investigate the opportunities for using GPUs to perform numerical integration for finite element simulations. The degree of concurrency and memory usage patterns are analysed for different types of finite element approximations. The results of numerical experiments designed to test execution efficiency on GPUs are presented. We draw some conclusions concerning advantages and disadvantages of off-loading numerical integration to GPUs for finite element calculations.

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ISOGEOMETRIC ANALYSIS IN PLASMA PHYSICS AND ELECTROMAGNETISM

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KEYWORDS: Isogeometric Analysis, Plasma Physics, Electromagnetism, Fast IGA

In this work, we will present some applications of the *IGA* approach in Plasma Physics simulation and electromagnetism, specially the time domain problem. To this purpose, we have developed a Python library, namely *PyIGA*.

In many problems in Plasma Physics, e.g. applications in the ITER project, we must deal with the complexity of the tokamak geometries. IGA seems to be an excellent approach to treat these problems. We have studied the gyrokinetic quasi-neutrality equation, [1], and also some *MHD* equilibrium problems, [2].

For the Maxwell time domain problem, we developed, [3], a new formulation of the exact sequence of Finite Element spaces based on splines, introduced by Buffa et al. [4], having the same properties as the Whitney Finite Element spaces traditionally used for the Finite Element solution of Maxwell's equations. As with the Whitney elements, one of Ampere's or Faraday's law can be discretized with a relation between the spline coefficients of the electric and magnetic fields independent of the topology of the mesh. The metric comes in through a discrete Finite Element Hodge operator which appears as the mass matrix involved in the other equation. This method allows us to inverse only one matrix at each time step.

We propose also a new strategy, namely the Fast IGA approach, specific to a large variety of domains, to solve some important problems: Maxwell's equations, current-hole problem, and more generally any problem where we need to inverse the mass matrix at each time step. In Figure 1, we present the typical CPU time needed for the Poisson's equation on a ring domain.

Spline degree	FIGA	SPLU
1	0.012	0.013
2	0.014	0.046
3	0.014	0.073
4	0.013	0.098
5	0.015	0.124
6	0.015	0.152
7	0.015	0.179

Spline degree	FIGA	SPLU
1	0.008	0.38
2	0.013	3.81
3	0.012	10.69
4	0.016	19.17
5	0.017	31.95
6	0.020	47.01
7	0.023	65.00

Figure 1: CPU-time, in seconds, spent in solving (left) and initializing (right) the linear system, using the new approach, namely Fast IGA, compared to SuperLU. Test done on a grid 128×128

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Shape sensitivity analysis based on isogeometric analysis applied to electromagnetic problems

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KEYWORDS: shape sensitivity, electromagnetics, material derivative, isogeometric analysis

Shape sensitivity analysis is an important tool for boundary shape optimization and numerous other applications. Uncertainty quantification, especially during design process, and error estimation can be cited here among others. Due to their great flexibility, Finite Element Methods are a standard choice for this purpose. As an accurate boundary description is important for the quality of the shape sensitivity analysis, isogeometric analysis [1] using non-uniform rational B-splines (NURBS) for the representation of the boundary is a very promising approach. NURBS are most widely used in CAD systems and the geometry representation can therefore be considered as exact. Considerable research efforts have been devoted to the shape sensitivity analysis based on isogeometric analysis, especially in the context of shape optimization [2].

In this work a formal approach to shape sensitivity analysis for electromagnetic problems and their discretization in the context of isogeometric analysis given in [3] is described using a perturbation of identity mapping and the concept of material (total) derivative. In order to account for the peculiarities of electromagnetism, the definition of the material derivative, following [4], will incorporate the transformation behavior of electromagnetic fields. Although motivated from differential geometry, the approach will be presented in terms of classical vector calculus.

The material derivative is chosen because of its favorable regularity properties compared to the shape (partial) derivative [5]. Furthermore, it allows for a common treatment of the continuous (differentiating, then discretizing) and the discrete (discretizing, then differentiating) approach to sensitivity analysis, introduced for a model elliptic problem in [6]. Explicitly differentiating the discretized system, which is often tedious, can thus be avoided. The extension of this approach to electromagnetics is a feature of the current work.

Regarding the shape sensitivity analysis of fields, numerical examples in two dimensions for the approximation of the material derivative are provided. These examples are used to validate the derived formulas. The convergence of the numerical schemes will be shown for mesh refinement and k-refinement. Concerning shape optimization, the shape sensitivity analysis of an energy functional, involving the adjoint state, will equally be considered. The accuracy of the results, due to the geometry representation and the good approximation properties of B-splines, show that isogeometric analysis is a well suited tool for shape sensitivity analysis in electromagnetics.

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Isogeometric collocation techniques for static and dynamic elasticity problems

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Abstract

In the framework of NURBS-based isogeometric analysis (see, e.g., [1, 2]), collocation techniques have been recently proposed in [3] as an interesting high-order low-cost alternative to standard Galerkin approaches. In this work, the results shown in [3] are extended to the case of linear elasticity. Particular attention is devoted to the imposition of boundary conditions (of both Dirichlet and Neumann type) and to the treatment of the multi-patch case. Also, the construction of explicit high-order (in space) collocation methods for elasto-dynamics is considered and studied.

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CONVERGENCE OF A DISCONTINUOUS GALERKIN SCHEME FOR TIME DOMAIN MAXWELL'S EQUATIONS IN A DISPERSIVE MEDIA

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KEYWORDS: Maxwell's equations, Dispersive media, DG methods, Numerical analysis.

We are interested in studying the propagation of electromagnetic waves through human tissues. This phenomenon arises in several biomedical areas, ranging from breast imaging, hyperthermia (technique used to kill cancer cells) to electroporation. In these situations, one has to face complex geometries and heterogeneous media. Thus in order to assess the biological effects of these techniques or to design adapted devices, there is a need of efficient numerical modeling techniques.
The modeling of the interaction of electromagnetic waves with human tissues relies on Maxwell's equations in a dispersive medium. Indeed human tissues contains a high percentage of water; this characteristic makes this medium dispersive. In such a medium, the speed of the wave propagating depends on the frequency : this takes the form of a frequency dependent permittivity prescribed by some known laws.
This study focuses on numerical aspects of this modeling with the mixed time domain form of Maxwell's equations. For a long time, such studies relied on Finite Differences Time Domain (FDTD) techniques (see eg. [4,5]) and despite the intrinsic limitation of this method with regards to the treatment of complex geometrical features, it is still widely used. More recently Finite Elements Time Domain (FETD) have also been investigated for this purpose (see eg. [2,3]), mostly from the theoretical point of view.
However, in the prospect of designing higher order numerical schemes, the Discontinuous Galerkin framework seems to be adapted. Less work has been done on this topic, but it now begins to be an active field for the case of dispersive media (see eg. [6]).
In this work we would like to go further in the numerical analysis. Following [1], we present a Discontinuous Galerkin framework for Maxwell's equations that we extend to dispersive media. The dispersive nature of the tissues will be modelized by a Debye medium and the effect taken into account via an Auxiliary Differential Equation: this takes the form of an ODE coupled to the system of Maxwell's equations, describing the evolution of the polarization. This work includes theoretical results concerning existence of solutions and the convergence analysis of both semi-discrete and fully discrete scheme, where the time discretization is performed by using a second order Leap-Frog scheme in time. Finally we present some preliminary 2D numerical results in order to validate the theoretical findings.

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EXTRACTING GENERALIZED FLUX INTENSITY FUNCTIONS ALONG CIRCULAR SINGULAR EDGES

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KEYWORDS: Flux/Stress intensity functions, Penny-shaped crack, 3-D singularities

This work is aimed at the computation of the edges flux/stress intensity functions (EFIFs/ESIFs) associated with the singular solution of elliptic equation in the vicinity of 3-D circular singular edges. These are of significant engineering importance in cracked and V-notched structures, in which the ESIFs may (and often do) vary along the crack front.

Herein we follow the methods presented in [1] to explicitly determine the solution to the Laplace equation in the vicinity of a circular singular edge in a general 3-D domain, by expanding the 3-D straight singular edge solution. This solution can be expressed in the form of an asymptotic series involving primal functions and two levels of shadow functions as follows (see [3]):

$$\tau = \sum_{\ell=0}^{\infty} \sum_{k=0} \partial_\theta^\ell A_k(\theta) \rho^{\alpha_k} \sum_{i=0} \left(\frac{\rho}{R}\right)^{i+\ell} \phi_{\ell,k,i}(\varphi) \tag{1}$$

where R is the distance of the singular point from the center of the edge, ρ and φ are "polar" coordinates from the edge, and θ is the position along the edge. Explicit expressions for the primal and shadow eigen-pairs are provided in case of a penny-shaped crack for an axi-symmetric and non axi-symmetric situations. The explicit solution is then exploited, in conjunction with a variation of the quasi-dual-function method [1,2] to extract the series coefficients $A_k(\theta)$, called edge generalized flux intensity functions (EGFIFs), from p-FE solutions.

The extension of the quasi-dual-function method to circular edges is presented and numerical results are provided where the EGFIFs are extracted for various example problems for an axisymmetric case and non-axisymmetric cases. This is a first step towards the computation of the Edge Stress Intensity Function (EGSIFs) in elasticity.

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ADDITIVE SCHWARZ DECOMPOSITION METHODS APPLIED TO ISOGEOMETRIC ANALYSIS

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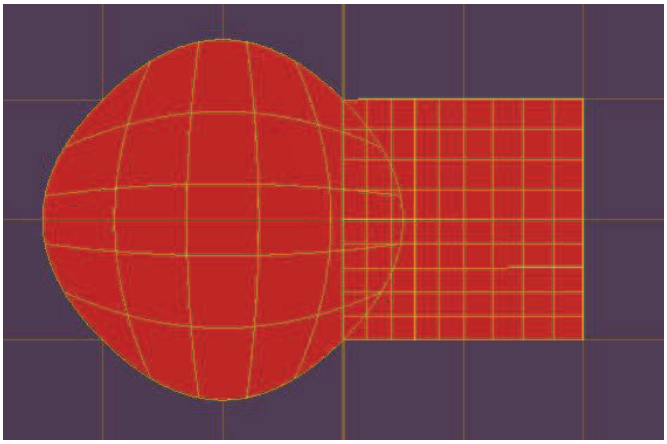
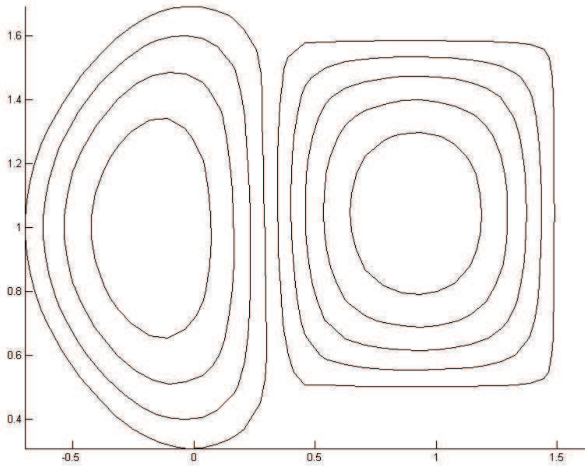
KEYWORDS: isogeometric analysis, domain decomposition, local refinement

One of the important aspects of IGA is its link to CAD geometry methods. Constructive Solid Geometry (CSG) represents the analysis domain as Boolean constructions from a set of primitives. In the present work we propose to use these primitives directly and apply Additive Schwarz on overlapping domains to actually solve the Isogeometric Analysis numerical problem: we iterate on a collection of overlapping domains, each defined by its own Isogeometric mapping. We consider the simple case of a Laplacian as well as linear elasticity.

For the Laplacian one can show on simple domains that, while the global stiffness matrices are not monotone, a ”weak maximum principle” still holds, thus warranting the convergence of the Additive Schwarz method without need for any preconditioning. Numerical examples show fast convergence, even on very distorted domains (where the Jacobian of the Isogeometry transformation is nearly singular). Numerical examples on simple elasticity problems also converge in a few iterations. Thus our approach can be applied to complex domains without having to use ”multipatch” constructs.





Another application of the present method is to use the ”Chimera” or zooming method described in [1] to do local refinements. This can be an alternative to more complex refinement methods, moreover it has the advantage of staying local. We will give examples of local ”zooming” instead of refinements. Trimmed volumes can also be treated this way applying the standard original Schwarz alternating method. In conclusion we show that these iterative methods can avoid the usage of complex construct such as T-Splines. Morover the additive methods are naturally parallels and thus can extend to large 3D problems.

Interestingly this approach brings also to the forefront some non simple operations that have to be solved on tri-variate volumes such as surface of intersection inside a volume projection of functions on such surfaces etc Part of it is done using the Irit[3] geometry system. We will give numerous simple examples all based on the use of the Geopdes software.

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A THREE-FIELD DUAL-MIXED *hp* FINITE ELEMENT MODEL FOR CYLINDRICAL SHELLS

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KEYWORDS: three-field, dual-mixed, dimensional reduction, cylindrical shell, finite element model

Finite element models based on dual-mixed variational principles can provide better convergence rates and higher accuracy for stresses than strain energy-based primal-mixed, or conventional displacement-based formulations. The displacement-based methods can lead to especially poor numerical results for incompressible materials and bending dominated thin plate and shell problems (incompressibility locking, shear- and membrane locking) [1, 2]. Stress-based finite element models have been proven to be locking-free in many cases and can give reliable numerical solutions, especially for the computed stresses.

One of the possibilities for the derivation of dimensionally reduced complementary energy-based shell models is to satisfy the translational and rotational equilibrium equations in a weak sense, using the displacements and rotations as Lagrangian multipliers. This procedure leads to the three-field dual-mixed variational principle of Hellinger–Reissner with independently approximated displacements, rotations and not *a priori* symmetric stresses [3, 4]. In the linear theory of elasticity its functional takes the form

$$\mathscr{HR}_d\left(\sigma^{rs},\varphi^s,u_p\right) = -\int\limits_{(V)} \frac{1}{2} C^{-1}_{pqrs}\sigma^{pq}\sigma^{rs}\mathrm{d}V + \int\limits_{(S_u)} \widetilde{u}_p\,\sigma^{pq} n_q \mathrm{d}S - \int\limits_{(V)} \left[u_p(\sigma^{pq}_{\ ;q}+b^p) - \sigma^{pq}\varepsilon_{pqs}\varphi^s\right]\mathrm{d}V\,, \tag{1}$$

where σ^{rs} is the non-symmetric stress tensor, φ^s is the axial vector of the skew-symmetric rotation tensor $\phi_{pq} = -\varepsilon_{pqs}\varphi^s$ with ε_{pqs} being the third-order permutation tensor and u_p is the displacement vector. The volume of the body is denoted by V, its boundary surface is S with outward unit normal n_q ($S = S_p \cup S_u$, $S_p \cap S_u = \emptyset$), b^p stand for the body forces and $\widetilde{u}_p$ are the prescribed displacements on the surface part S_u. The fourth-order tensor $C^{-1}_{pqrs} = C^{-1}_{pqsr} = C^{-1}_{rspq}$ is the elastic compliance tensor. The only subsidiary conditions to functional (1) are the stress boundary conditions

$$\sigma^{kl} n_l = \widetilde{p}^k \quad \text{on} \quad S_p, \tag{2}$$

where $\widetilde{p}^k$ are prescribed surface tractions on S_p.

A dual-mixed *hp* finite element model with stable polynomial stress- and displacement interpolation and C^0 continuous normal components of stresses is constructed for cylindrical shells. Two important properties of the shell model are: (i) classical kinematical hypotheses regarding the deformation of the normal to the shell middle surface are not applied and (ii) unmodified three-dimensional linear stress-strain relations are applied. The numerical results are compared to the analytic solutions of Koiter's cylindrical shell model. The convergences in the energy norm as well as in the maximum norm of stresses and displacements are rapid for both *h*- and *p*-extension, even if the Poisson ratio is close to 0.5, i.e., the dual-mixed *hp* finite element model is free from incompressibility locking. The shell model and element developed give accurate and reliable numerical results not only for thin but also for moderately thick cylindrical shells.

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RELIABLE PATIENT-SPECIFIC p-FEM SIMULATION OF FEMUR'S MECHANICAL RESPONSE

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KEYWORDS: p-FE, qCT, femur, bone biomechanics

A reliable subject-specific FE model to be used in clinical practice requires a high level of automation, verification and validation. Herein we present recent results for generating p-FE models based on quantitative computed tomography (qCT) scans. Femur's geometry is represented by accurate smooth surfaces based on which a p-FE auto-mesh is generated. Inhomogeneous linear elastic material properties at each region (cortical and trabecular) are assigned to the FE models directly from the CT scan at each integration point $E(x,y,z)$. In addition orthotropic material properties determined by micromechanics-based methods were also considered in the p-FE model and their influence on the mechanical response is examined [1].

After verification of the numerical results we validate these on a cohort of 17 fresh-frozen femurs which were defrosted, qCT-scanned, and thereafter tested in an in-vitro setting [2,3]. p-FE models mimicking the experiments condition were created from the qCT-scans and the computed displacements and strains were compared to the measured values in the experiments. The FE results correlated with the experimental observation by linear regression with $R^2 = 0.97$ and a slope of 0.96 when comparing both displacements and strains $(n = 263)$. The encouraging results enable us to enhance the research to other human bones like metatarsal, humerus, and vertebrae. This study exemplifies that the presented method is in an advanced stage to be used in clinical computer-aided decision making.

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ISOGEOMETRIC SIMULATION OF SOME REAL ELECTROMAGNETIC APPLICATIONS

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KEYWORDS: isogeometric analysis, induction heating

After the introduction of isogeometric analysis (IGA) in [1], the research on isogeometric discretizations has grown up considerably. Apart from the possibility of improving the efficiency of communication between CAD software and PDE solvers (which is still a research topic), the main advantage of IGA with respect to finite elements (FEM) seems to be the higher continuity of the discrete solutions. This is already known to provide better convergence in terms of the degrees of freedom, better stability properties, and less numerical dispersion.

The goal of this work is to understand the behavior of IGA in the simulation of some real electromagnetic devices, and to check whether the method provides in computational electromagnetism the same advantages that it has already shown in computational mechanics. We have chosen for this task two problems for which a FEM simulation was already available, in order to compare the given results.

The first problem we analyze concerns the computation of per unit length parameters (resistance and inductance) in power electrical cables. The main difficulty for the computation with finite elements arises at high frequencies, when the skin effect becomes more important, since a very fine mesh is needed close to the boundary. We will provide a comparison of the results both with FEM and IGA, and also with the method with high order surface impedance boundary conditions used in [2], which is known to work well at high frequencies.

The second problem is the simulation of an induction heating furnace, as the one given in [3]. The furnace consists of a helical inductor, the material to be treated, and a crucible that contains it. The simulations carried out in [3] involve electromagnetic, thermal and hydrodynamic effects, since the material is heated until melting. For the present work we have developed the thermoelectrical simulation with an isogeometric method. This problem already presents some interesting features, as the skin effect in the electromagnetic conductors, or the steep temperature gradients on the interfaces between thermal conductors and insulators.

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